A Regularized Framework for Multi-Channel Modal Analysis

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Overview

- Modal analysis of ringdowns is frequently conducted using single-channel techniques, such as Prony's method, which possess the following drawbacks:
  - A different set of estimated eigenvalues is returned for each signal.
  - It can be difficult to distinguish between nearby modes with different shapes.
- Existing multi-channel algorithms offer limited control over the variance of eigenvalue estimates in frequency and damping.
- We develop an optimization-based multi-channel method that provides greater control over the spectral properties of the model fit to the data.

Optimization formulation

- Recall that the discrete-time state update equation is

$$x_{k+1} = Ax_k + Bu_k, \tag{1}$$

which reduces to $x_{k+1} = Ax_k$ in the free response (i.e., when $u_k = 0$).

- Formulation:

$$\underset{A}{\text{minimize}} \quad \sum_{k \in \mathcal{K}} \left\| x_{k+1} - Ax_k \right\|_2 + \mu \left\| A \right\|_*. \tag{2}$$

- Notation: $\|A\|_*$ returns the sum of the singular values of A, which is a useful heuristic for $\operatorname{rank}()$.
  - The parameter μ can be thought of as a tuning knob for adjusting the number of oscillatory modes present in the model.

Monitored locations

- Modal analysis of ringdowns using data collected from 26 simulated sensors.

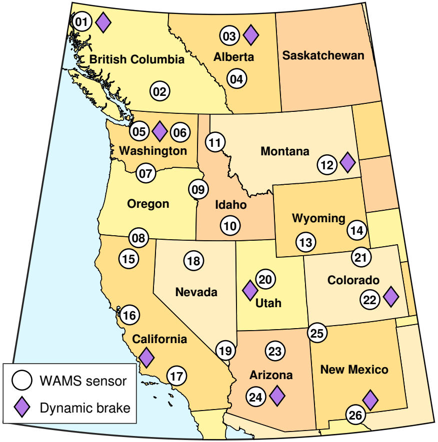


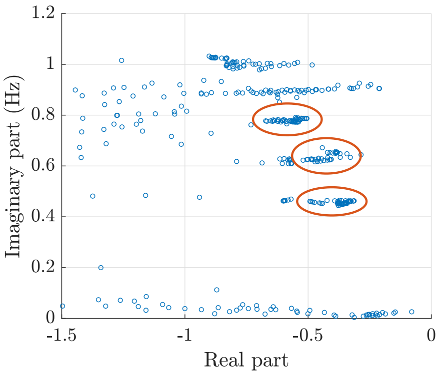
Figure: Points of measurement (simul.).

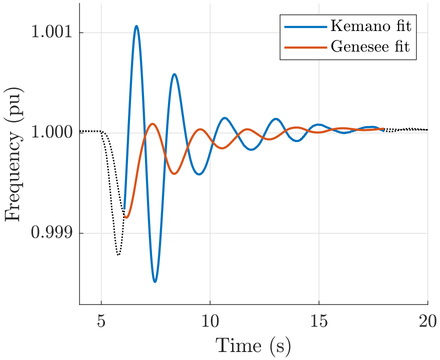
Figure: Monitored Locations

No.	Name	No.	Name
1	Kemano	14	Laramie
2	Nicola	15	Round Mt.
3	Genesee	16	Tesla
4	Langdon	17	Vincent
5	Monroe	18	Valmy
6	Coulee	19	Mead
7	Big Eddy	20	Mona
8	Malin	21	Ault
9	Brownlee	22	Comanche
10	Midpoint	23	Moenkopi
11	Taft	24	Hassayampa
12	Colstrip	25	Four Corners
13	Bridger	26	Newman

Chief Joseph Brake Example

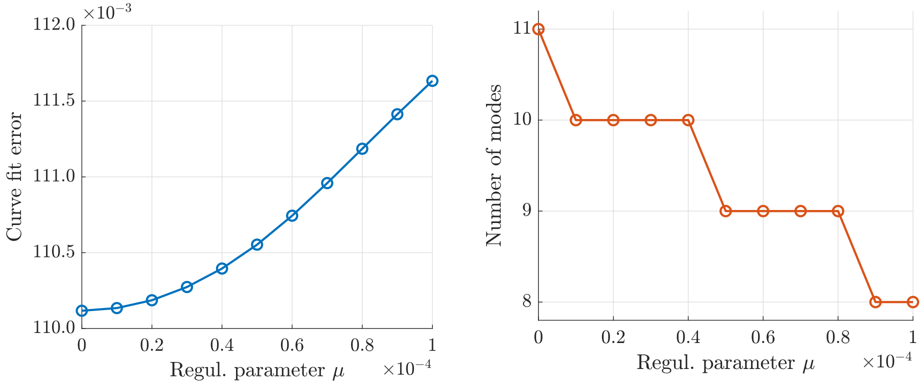
- We explore the Pareto frontier by varying μ and the curve fitting window.
- The ellipses that bound the mode estimates are smaller when $\mu > 0$.





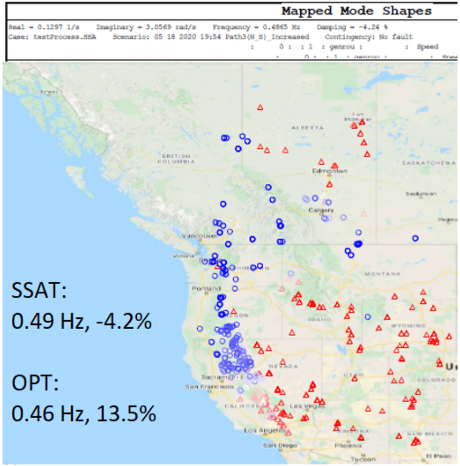
The effect of sweeping μ

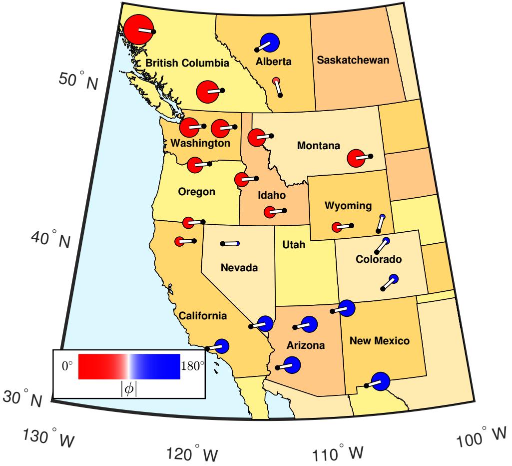
- Two key things happen as μ increases:
  - The curve fit accuracy gets slightly worse (expected due to trade off).
  - The number of oscillatory modes (complex pole pairs) decreases.



North-South B mode comparison

- Working with CAISO, we made some preliminary comparisons with the commercial software package SSAT (our results are labeled “OPT”).





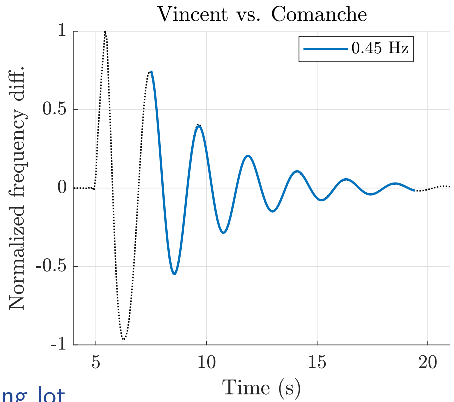
Summary

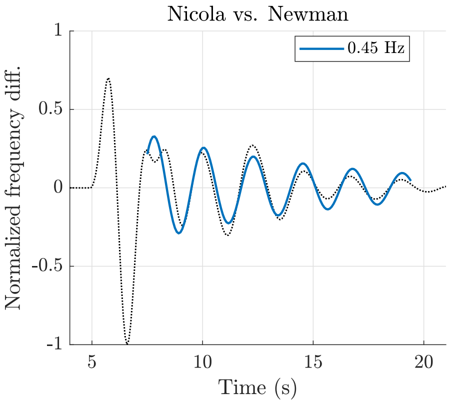
- We are applying these techniques within the WECC WIMRG.
- The group plans to release an updated version of the WECC white paper describing the properties of the inter-area modes.
  - These techniques are being used to generate eigenvalue estimates and mode shapes for the report.
- We have identified intriguing new possibilities that could improve our understanding of the system.
  - Analysis indicates that what has historically been referred to as the BC Mode may actually be more than one mode.

Parking lot

Time-domain classification of NS-B/EW-A modes

- For a brake insertion near Comanche, we observe something strange in the Nicola v. Newman frequency difference.
- The damping appears much lower, but in reality is a result of interaction between the NS-B and EW-A modes.





Time-domain verification of BC Mode B

- In Light Spring, Colstrip and Tesla are in phase for BC Mode A, but out of phase for BC Mode B (dominant modal component).

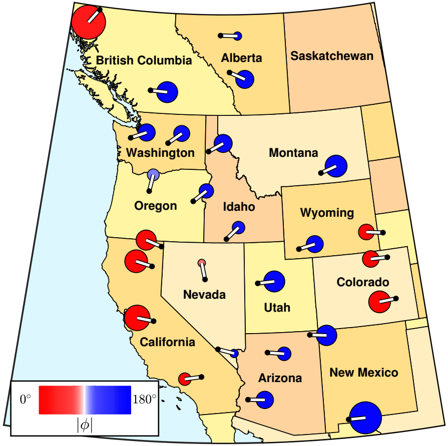


Figure: Light Spring, 0.69 Hz, 11.4 %.

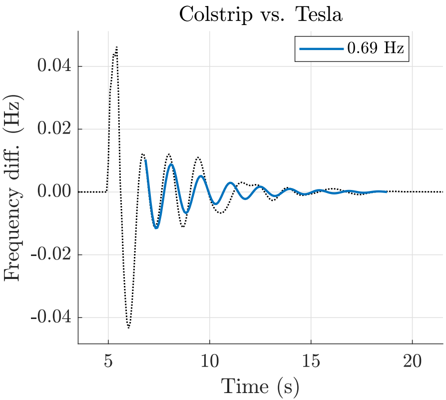


Figure: Ringdown for brake insertion near Diablo Canyon.