Tracking Three Phase Untransposed Transmission Line Parameters Using Synchronized Measurements

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Transmission line parameters may not all be accurate in the existing data base.

Only positive and zero sequence parameters may be available, yet not all lines are fully transposed.

Parameters may change due to changes in ambient temperature, wind, loading etc.

A method to track the parameters of the fully coupled three-phase line model is needed!
Consider the following untransposed three-phase transmission line with mutual coupling between phases:

Assume PMU measurements at both terminals of the line.
Nodal equations for the line can be written as:

\[ \tilde{I} = Y \tilde{V} \]

Considering the pi-model for the line:

\[ V_s = \begin{bmatrix} V_{sa} \\ V_{sb} \\ V_{sc} \end{bmatrix}, \quad I_s = \begin{bmatrix} I_{sa} \\ I_{sb} \\ I_{sc} \end{bmatrix}, \]
\[ V_m = \begin{bmatrix} V_{ma} \\ V_{mb} \\ V_{mc} \end{bmatrix}, \quad I_m = \begin{bmatrix} I_{ma} \\ I_{mb} \\ I_{mc} \end{bmatrix}, \]
\[ \tilde{V} = \begin{bmatrix} V_s \\ V_m \end{bmatrix}, \quad \tilde{I} = \begin{bmatrix} I_s \\ I_m \end{bmatrix}. \]

\( Y \) will then be a 6-by-6 matrix (complex entries) given below:

\[
Y = \begin{bmatrix}
Y_{shunt}/2 + Z_{series}^{-1} & -Z_{series}^{-1} \\
-Z_{series}^{-1} & Y_{shunt}/2 + Z_{series}^{-1}
\end{bmatrix}
\]
How many independent parameters will be in an untransposed transmission line model?

\[ Z_{\text{series}} = \begin{bmatrix} Z_{aa} & Z_{ab} & Z_{ac} \\ Z_{ab} & Z_{bb} & Z_{bc} \\ Z_{ac} & Z_{bc} & Z_{cc} \end{bmatrix} \]

\[ Y_{\text{shunt}} = \begin{bmatrix} Y_{aa} & Y_{ab} & Y_{ac} \\ Y_{ab} & Y_{bb} & Y_{bc} \\ Y_{ac} & Y_{bc} & Y_{cc} \end{bmatrix} \]

- Impedance \( Z_{\text{series}} \) consists of line resistance and inductance.
- Admittance \( Y_{\text{shunt}} \) consists of shunt capacitance and conductance.
- For an overhead line, conductance is commonly ignored, i.e. \( Y_{\text{shunt}} \) will just have an imaginary part.

6 complex unknowns in \( Z_{\text{series}} \), 6 imaginary unknowns in \( Y_{\text{shunt}} \)

Total: \( 2 \times 6 + 6 = 18 \) real unknowns, make up the unknown vector \( p \)
Using rectangular coordinates:

\[
\tilde{I} = Y \tilde{V}
\]

\[
I = H_p V
\]

Equation \( I = H_p V \) can be rearranged as \( I = H_V p \) where:

- \( I \) is a 12-by-1 vector that contains the current measurements in rectangular coordinates from both terminals of the line.
- \( p \) is the 18-by-1 unknown parameter vector.
- \( H_V \) is the 12-by-18 rearranged coefficient matrix consisting of measured voltages at the line terminals.
Regarding the newly formulated equation $I = H_V p$:

- Good news: This is a linear equation!
- Bad news: More unknowns $p$ (18) than measurements $I$ (12)

**QUESTIONS**

- Can we use multiple measurement snapshots? **NO**
  - $H_V$ will still have low rank because voltages will not change much between measurement scans

- Can we model parameter dynamics? **YES**
  - Assumed dynamics: $p_{k+1} = p_k + w_{p,k}$ (process noise)
Parameter Tracking Formulation

- Parameter dynamics: \( p_{k+1} = p_k + w_{p,k} \)
- Measurement equation: \( I_k = H_{V,k} p_k + v_k \)
  - \( k \) is the time instant
  - Current measurement noise is modeled as \( v_k \)
- Use Kalman filter to solve the parameter tracking problem

ONE MORE QUESTION:

- Can we use the measured voltages directly in \( H_{V,k} \)? NO
  - Measurements always contain error or noise.
Therefore, we introduce a three phase static state estimator using PMU measurements to estimate the states $x$

$$
\begin{bmatrix}
V \\
I
\end{bmatrix} = 
\begin{bmatrix}
\text{Identity matrix} \\
H_p
\end{bmatrix} x + e
$$

- $V$ is voltage measurement in rectangular form, it is a 12-by-1 vector.
- $I$ is current measurement in rectangular form, it is a 12-by-1 vector.
- $x$ is the state in rectangular form, it is still a 12-by-1 vector.
- $H_p$ is the coefficient matrix consists of parameters, 12-by-12 matrix.
- $e$ is the measurement noise.

Use Least-Squares to solve this state estimation problem.

But the parameters in $H_p$ are not known!
We want to estimate states from parameters and current measurements

We also want to estimate parameters from states and current measurements

Solution:

Iterative between state estimation and parameter tracking problems
Convergence of the iterations

- It is important to ensure that iterations always converge!

- When two successive iterations yield close enough solutions, iterations can be terminated and results will be trusted.

- Trace of the error covariance of parameters will be used to monitor the “health” of iterations:
  - Error covariance is one of the major measures of estimation accuracy
    - The smaller, the better
  - Trace of error covariance for this problem is always convex, so when it begins to increase, iterations can be terminated.
We have tested the algorithm on several simulated cases.
Although some simulated cases are not realistic in practice, they justify the algorithm since the actual values of parameters are available and the estimates can track them almost perfectly.

Varying parameters with abrupt changes
Test our algorithm on a real transmission line with two PMUs on both sides.

Duration is 5.5 minutes, have about 9800 data points.

CPU running time < 2ms per data point.

The estimated parameters are close to values in data base.

Even small variations of the parameters can be tracked accurately.
Advantages

- Can obtain all the parameters of a three phase (untransposed) transmission line
  - Not only positive sequence, but also negative and zero sequence data

- Can track line parameters dynamically using limited information

- Can be implemented for large scale systems with sparsely installed PMUs.
Applications

- Validate database for transmission line parameters
- Track changes of line parameters dynamically
- Support state estimator, especially for three phase phasor only state estimator, dynamic state estimator etc.
- Accurate operation/setting of protective relays and dynamic relays
- Monitoring line corona loss under different weather conditions
THANK YOU!