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Phase Angle Formulation

• PMU reports measured values time-tagged to UTC
  • Amplitude
  • Phase Angle
  • Frequency
  • Rate of change of frequency (ROCOF)

• Signal can be modeled as \( x(t) \) and the Synchrophasor is calculated relative to the nominal system frequency synchronized to UTC:

\[
x(t) = X_m(t) \cos(2\pi f_0 t + \varphi(t))
\]

• The synchrophasor is: \( X(t) = (X_m(t)/\sqrt{2})e^{j(\varphi(t))} \)
  • The synchrophasor includes all the dynamic changes in amplitude and phase angle
  • Off-nominal frequency appears as a change in phase angle
Frequency & Angle Example

• Off-nominal frequency $f = f_0 + \Delta f$:

• Signal formula is: $x(t) = X_m \cos(2\pi(f_0 + \Delta f) t + \phi) = X_m \cos(2\pi f_0 t + 2\pi \Delta f t + \phi)$$\phi(t)$

• Then the synchrophasor is: $X(t) = X_m/\sqrt{2} e^{j(2\pi \Delta f t + \phi)}$

• Constant $f$ off-nominal results in angle increasing/decreasing linearly
PMU Window Measurement

• A PMU examines the signal over short time (measurement window)
  • Generally a few cycles

• It estimates the phase angle at each point and resolves the answer to a principal value $\theta$ where $-\pi < \theta \leq \pi$.
  • The PMU only estimates phasor angle, it does not track time error
  • In a time plot, PMU may appear to “wrap” but that is not something a PMU does... It is an artifact of the way we resolve angle to the principal value
Accumulated Phase

- As frequency varies off-nominal, phase angle will vary
  - System frequency > nominal, the phase angle increases
  - System frequency < nominal, the phase angle decreases
  - Phase angle may decrease for period of time and then increase for some time

- Accumulated phase can be tracked based on some starting point
The Issues

• To calculate phase angle between two measurements, we have to be sure they do not span the 180° rollover

• As they rotate with off-nominal frequency, they do not roll-over at the same time and create a spike in the measurement

• We may want to record the accumulated system time error through the accumulated phase angle
Solution approaches

• Each time angle differences are calculated, resolve them into their principal value (subtract then unwrap)

• Monitor each angle from some starting point and add the full circle (360°) to an accumulated phase angle value each time it rolls over (unwrap the angle). Calculate all angle differences from these unwrapped angles
  • Starting points have to be unwrapped initially

• Both approaches assume there will be no real angle differences that are greater than 180°. In fact, given any two angles, it is impossible to determine angle differences >180° without additional information
  • Angles at intermediate points
  • Flow calculations with equipment and line parameters (state estimation)
Angle Wrapping Definition

\[ \Omega(\varphi) = \begin{cases} 
\varphi - 2n\pi, & \text{where } n \in I \text{ is an integer so that } -\pi < \varphi - 2n\pi < \pi \smallskip 
\pi, & \text{where } n \in I \text{ is an integer so that } \varphi - 2n\pi = \pi 
\end{cases} \]
Angle Unwrapping Definition

\[
\begin{align*}
\Psi(\phi(t_0)) &= \phi(t_0), \\
\Psi(\phi(t_k)) &= \phi(t_k) + \Psi(\phi(t_{k-1})) - \phi(t_{k-1}) + 2n\pi, \text{ where } n \text{ is an integer that minimizes } |\phi(t_k) - \phi(t_{k-1}) + 2n\pi|, \forall k \neq 0.
\end{align*}
\]
Unwrap Then Subtract
Subtract Then Wrap

double UnwrapAngleDelta(double AngleDelta)
{
    double rval = AngleDelta;
    if (Math.Abs(AngleDelta) > 180)
    {
        if (Math.Sign(AngleDelta) < 0)
            // is AngleDelta negative?
        {
            rval = AngleDelta + 360;
        }
        else // AngleDelta is positive
        {
            rval = AngleDelta - 360;
        }
    }
    return (rval);
}
Vector Math

This approach lets the computer math formulation resolve the angle. This works because vector (and complex number) math works in rectangular coordinates which do not have a roll over and resolves the answer to the principal value.

\[ \alpha_{Diff} = \angle (\overline{V_1} \cdot \overline{V_1}^*) \]

\[ V1 = 1 + 0i; \]
\[ V2 = 1 + 1i; \]
\[ dDiff = \text{angle}(V1*\text{conj}(V2))*180/\pi \text{  \% in degrees} \]
## Phase Angle Difference Calculation Considerations

<table>
<thead>
<tr>
<th></th>
<th>Simple Arithmetic Subtraction</th>
<th>Unwrap Then Subtract</th>
<th>Subtract Then Wrap</th>
<th>Vector Mathematics</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>EMS Calculation Capabilities</strong></td>
<td>Achievable</td>
<td>EMS computation limitations</td>
<td>Achievable</td>
<td>Not possible</td>
</tr>
<tr>
<td><strong>Calculation Accuracy</strong></td>
<td>Poor, when raw angles wrap</td>
<td>Accurate calculation. Same level of accuracy.</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Calculation Speed</strong></td>
<td></td>
<td>Fast</td>
<td>Slow</td>
<td></td>
</tr>
<tr>
<td><strong>CPU Usage</strong></td>
<td>Minimal</td>
<td></td>
<td></td>
<td>Minimal</td>
</tr>
<tr>
<td><strong>Data Storage</strong></td>
<td>Minimal</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Missing Data Issues</strong></td>
<td>Minimal issues; only calculations with missing data voided</td>
<td>Missing data can result in wrap around points getting missed</td>
<td>Minimal issues; only calculations with missing data voided</td>
<td>Minimal issues; only calculations with missing data voided</td>
</tr>
<tr>
<td><strong>Angle Resetting</strong></td>
<td>None</td>
<td>User can specify rate of resetting⁴</td>
<td>None</td>
<td>None</td>
</tr>
<tr>
<td><strong>Application Requirements</strong></td>
<td>Not acceptable</td>
<td>Acceptable</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Absolute Time Error Correction</strong></td>
<td>Not achievable</td>
<td>Required</td>
<td>Not achievable</td>
<td>Not achievable</td>
</tr>
</tbody>
</table>
Accumulated Phase Angle Difference

- Phase angle reported by a PMU is defined as the principal value (PV) of the signal relative to the reference waveform synchronized to UTC.
- PMU is not designed to keep track of evolution of relative phase of two signals since some arbitrary time zero.
- A PMU could be built to track accumulated phase
  - The user would have to establish a start time
  - Accumulated phase can become arbitrarily large, so a reset mechanism is required
  - Phasor value reporting would need modification to report very large values
- Accumulated phase can be calculated or obtained from the PMU signals by “unwrapping” the measurement over a period of time.
- This is effectively an alternative to calculating system time error used in SCADA systems
Phase Angle Distance

Shows how phase angle can increase to large values over time.
Rollover Counter (ROC)

For keeping track of the number of roll-overs that have occurred
Concluding Remarks

• Applications must account for principal value of phase angle reported by PMU resolving to interval (+180,-180] in any calculation.

• Applications should document and clearly define angle wrapping and unwrapping they use for industry reference.

• Commonly used angle difference calculation methods explained in detail.
  • Vector math is “gold standard” – computationally complex but useful for offline applications.
  • “Subtract then Wrap” and “Unwrap then Subtract” both accurate and their pros and cons.
  • Simple arithmetic difference should be avoided.

• Considerations include speed, accuracy, CPU usage, data storage, missing data, etc.

• Phase Angle Distance and Rollover Counter (ROC) introduced.