Interarea Model Estimation for Large-scale Electric Power Systems using Synchronized Phasor Measurements

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Two-machine Equivalents

Our Approach:

Dynamic Measurements

Signal Separation

Interarea Oscillation

Model Identification & Reduction Problem
‘Interarea Model Estimation (IME)’
IME: Method

\[ \tilde{I} = I \angle \theta_1 \]

\[ \tilde{V}_1 = V_1 \angle \theta_1 \quad \tilde{V}_2 = V_2 \angle \theta_2 \]

**Problem:** How to estimate all parameters?

\[ x_1, x_2, H_1, H_2 \]

\[ \dot{\delta} = \omega \]

\[ 2 \frac{H_1 H_2}{H_1 + H_2} \dot{\omega} = \frac{H_2 P_{m1} - H_1 P_{m2}}{H_1 + H_2} - \frac{E_1 E_2}{(x_1 + x_e + x_2)} \sin \delta \]

*Swing Equation*
IME: Method (Reactance Extrapolation)

- **Key idea**: Amplitude of voltage oscillation at any point is a function of its electrical distance from the two fixed voltage sources.

\[
\tilde{V}(x) = [E_2(1-a) + E_1a \cos(\delta)] + j E_1a \sin(\delta), \quad a = \frac{x}{x_1 + x_e + x_2}
\]

- Voltage magnitude: \( V = |\tilde{V}(x)| = \sqrt{c + 2E_1E_2(a-a^2)\cos(\delta)}, \quad c = (1-a)^2E_2^2 + a^2E_1^2 \)

- Assume the system is initially in an equilibrium (\( \delta_0, \omega_0 = 0, V_{ss} \)):

\[
\Delta V(x) = J(a, \delta_0) \Delta \delta
\]

\[
J(a, \delta_0) := \left. \frac{\partial V(a, \delta_0)}{\partial \delta} \right|_{\delta = \delta_0} = \frac{-E_1E_2}{V(a, \delta_0)}(a-a^2)\sin(\delta_0)
\]
Reactance Extrapolation

\[ \Delta V(x) = \frac{-E_1 E_2}{V(a, \delta_0)} (a - a^2) \sin(\delta_0) \Delta \delta \]
Reactance Extrapolation

\[ \Delta V(x) = \frac{-E_1 E_2}{V(a, \delta_0)} (a - a^2) \sin(\delta_0) \Delta \delta \]

\[ \Delta V(x) V(a, \delta_0) = -E_1 E_2 \sin(\delta_0) (a - a^2) \Delta \delta(t) \]

can be computed from measurements

\[ V_n(x, t) = A (a - a^2) \Delta \delta(t) \]

solution of a linear differential equation
**Reactance Extrapolation**

\[ \Delta V(x) = \frac{-E_1E_2}{V(a, \delta_0)} (a - a^2) \sin(\delta_0) \Delta \delta \]

\[ \Delta V(x)V(a, \delta_0) = -E_1E_2 \sin(\delta_0) (a - a^2) \Delta \delta(t) \]

Refer to as: ‘Normalized voltage’

Note: **Spatial** and **temporal** dependence are separated

- Fix time: \( t=t^* \)

\[ V_n(x, t^*) = A (a - a^2) \Delta \delta(t^*) \]

How can we use this relation to solve our problem?
Reactance Extrapolation

\[ V_n(x,t^*) = A (a - a^2) \Delta \delta(t^*) \]

At Bus 1,

\[ \delta \Delta - \frac{1}{2} \left( \frac{V_{Bus1}}{V_{Busn}} \right) \]

At Bus 2,

\[ a_2 = \frac{x_2}{x_1 + x_e + x_2} \quad \rightarrow \quad V_{n, Bus2} = A \left( a_2 - a_2^2 \right) \Delta \delta(t^*) \]

\[ \frac{V_{n, Bus2}}{V_{n, Bus1}} = \frac{a_2(1 - a_2)}{a_1(1 - a_1)} \]

At Bus 1,

\[ a_1 = \frac{x_e + x_2}{x_1 + x_e + x_2} \quad \rightarrow \quad V_{n, Bus1} = A \left( a_1 - a_1^2 \right) \Delta \delta(t^*) \]

\[ \frac{V_{n, Bus3}}{V_{n, Bus1}} = \frac{a_3(1 - a_3)}{a_1(1 - a_1)} \]

• Need one more equation
  - hence, need one more measurement at a known distance
Reactance Extrapolation

\[ V_n(a) = A \, a \, (1 - a) \]

Key idea: Exploit the spatial variation of phasor outputs
IME: Method (*Inertia Estimation*)

- From linearized model
  \[ f_s = \frac{1}{2\pi} \sqrt{\frac{E_1 E_2 \cos(\delta_0) \Omega}{2H(x_e + x_1 + x_2)}} \]
  
  where \( f_s \) is the measured swing frequency and \( H = \frac{H_1 H_2}{H_1 + H_2} \)

- For a second equation in \( H_1 \) and \( H_2 \), use law of conservation of angular momentum
  \[
  2H_1 \omega_1 + 2H_2 \omega_2 = 2 \int (H_1 \dot{\omega}_1 + H_2 \dot{\omega}_2) dt = \int (P_{m1} - P_{e1} + P_{m2} - P_{e2}) dt = 0
  \]
  \[
  \Rightarrow \quad \frac{H_1}{H_2} = -\frac{\omega_2}{\omega_1}
  \]

- However, \( \omega_1 \) and \( \omega_2 \) are not available from PMU data,
  
  → Estimate \( \omega_1 \) and \( \omega_2 \) from the measured frequencies \( \xi_1 \) and \( \xi_2 \) at Buses 1 and 2
IME: Method *(Inertia Estimation)*

- Express *voltage angle* $\theta$ as a function of $\delta$, and differentiate wrt time to obtain a relation between the machine speeds and bus frequencies:

$$
\xi_1 = \frac{a_1 \omega_1 + b_1 (\omega_1 + \omega_2) \cos(\delta_1 - \delta_2) + c_1 \omega_2}{a_1 + 2b_1 \cos(\delta_1 - \delta_2) + c_1}
$$

$$
\xi_2 = \frac{a_2 \omega_1 + b_2 (\omega_1 + \omega_2) \cos(\delta_1 - \delta_2) + c_2 \omega_2}{a_2 + 2b_2 \cos(\delta_1 - \delta_2) + c_2}
$$

- $\xi_1$ and $\xi_2$ are measured, and $a_i$, $b_i$, $c_i$ are known from reactance extrapolation.

- Hence, we calculate $\omega_1/\omega_2$ to solve for $H_1$ and $H_2$.

where,

$$
a_i = E_1^2 (1-r_i)^2, \quad b_i = E_1 E_2 r_i (1-r_i),
$$

$$
c_i = E_2^2 r_i^2$$

![Plot showing normalized reactance and frequency](image-url)
Illustration: 2-Machine Example

- Illustrate DME on classical 2-machine model
- Disturbance is applied to the system and the response simulated in MATLAB

Voltage oscillations at 3 buses

Bus angle oscillations

Bus frequency oscillations

\[ G(s) = \frac{s}{sT + 1} \]

DME Algorithm

Exact values:
\[ x_1 = 0.34 \text{ pu}, \quad x_2 = 0.39 \text{ pu} \]

\[ V_{1m} = 0.0292 \quad V_{2m} = 0.0316 \quad V_{3m} = 0.0371 \]
\[ V_{1ss} = 1.0320 \quad V_{2ss} = 1.0317 \quad V_{3ss} = 1.0136 \]
\[ V_{1n} = 0.0301 \quad V_{2n} = 0.0326 \quad V_{3n} = 0.0376 \]

DME

Exact values:
\[ H_1 = 6.48 \text{ pu}, \quad H_2 = 9.49 \text{ pu} \]

\[ H_1 = 6.5 \text{ pu}, \quad H_2 = 9.5 \text{ pu} \]
IME for Complex System Topologies

- Intermediate voltage support

**Shunt Capacitance**

\[ \tilde{E}_1 = E_1 \angle \delta_1 \]

**Static VAr Compensation**

\[ \tilde{E}_1 = E_1 \angle \delta_1 \]

**Generator Support**
IME for Complex System Topologies

**Static VAr Compensation**

\[
\begin{align*}
\tilde{E}_1 &= E_1 \angle \delta_1 \\
\tilde{E}_2 &= E_2 \angle \delta_2 \\
1 &\quad 2 \quad 3 \\
\tilde{V}_1 &\quad \tilde{V}_c &\quad \tilde{V}_2 \\
1 &\quad jx_1 &\quad jx_{e1} &\quad jx_{e2} &\quad jx_2 \\
\tilde{I}_1 &\quad \tilde{I}_c &\quad \tilde{I}_2 \\
\end{align*}
\]

- \( \dot{B} = \frac{B}{\tau} + \frac{k}{\tau} (V_r - V_c) \), \( B = \frac{1}{x_c} \)
- \( B = k(V_r - V_c) \) (assuming \( \tau \) is small)
- \( \tilde{V}_c = f_3(E_1, E_2, \delta, x_{e1}, x_{e2}, x_c(V_c)) \)

\[ \text{solve for a quadratic in } V_c(\delta) \]

\[ B = k(V_r - V_c(\delta)) \]

\[ |\tilde{V}(x)| = f_4(E_1, E_2, \delta, x_{e1}, x_{e2}, x, B(\delta)) \rightarrow \text{extra terms in } V_n \]
Application to WECC Data

Needs processing to get usable data

- Sudden change/jump
- Oscillations
- Slowly varying steady-state (governor effects)
WECC Data

Band-pass Filter
Choose pass-band covering typical swing mode range

Oscillations

Quasi-steady State
• Can use modal identification methods such as: **ERA, Prony, Steiglitz-McBride**
Conclusions

• We developed novel methods for model identification and reduction of two-area power systems to represent interarea dynamics
  - spatial variation patterns of phasor variables are exploited

• Fast sampled *dynamic phasor measurements* are used for building these tools

• Both with and without voltage support cases are considered

• Appropriate signal processing tools are developed

• The method enables better estimation of energy margins, better estimation of wave speeds, easier design of PSS, etc.
Thank You