PMU-Based Monitoring of Power System Dynamics

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PMU-Based Power System Dynamics Monitoring

**Goal**
Wide area monitoring of rotor angle stability to prevent widespread blackouts.

**Methodology**
- State Calculator (SC).
- Maximal Lyapunov Exponent (MLE).
- Observability Analysis of Nonlinear Dynamic Systems.

**Flowchart**

State Calculator and MLE

Dynamic Model of a Power System
\[ \dot{x} = f(x) \quad \text{where} \quad x = [\theta, \omega] \quad \text{and} \quad f = h(\delta, \omega) \]

State Calculator
- Assumed \( x_i(0), x_j(0) \) are observed by PMU measurements.
- \( x_i(0), \ldots, x_k(0) \) are estimated by
  \[ \dot{x}(t + \Delta t) = x_i(t) + \Delta t \left( \frac{1}{2} f(x_i(t)) \right) \quad \text{for} \quad i = k + 1, \ldots, n \]

MLE
- The Lyapunov Exponents are used to characterize the exponential divergence or convergence of nearby trajectories.
- MLE is calculated by the approximation of:
  \[ \lambda(t) = \frac{\Delta \text{MLE}}{\Delta t} \]

- MLE of \( x \) is calculated to monitor rotor angle stability after a disturbance.
  - If MLE < 0, the system is (asymptotically) stable.
  - Otherwise, it is considered "unstable".

Nonlinear Dynamic Observability

A dynamic system is observable if, for any time \( t \), the current state \( x(t) \) can be determined using only the measurements \( h(x(t)) \).

- The system is locally observable at \( x_i \) if the matrix \( L_{h_i} \) has a full rank. \( L_{h_i} \) is called the observability matrix, in which
  \[ L_{h_i} = \begin{pmatrix} L_{h_{i1}} & \cdots & L_{h_{iN}} \end{pmatrix} \]

- The observability indices are chosen as an index to assess the level of system observability.

- Observability indices can be tracked along system trajectory.

179-Bus WECC System:

As the number of PMUs increases, the level of observability also improves.

PMU-Based and MLE

PMU

Generators with PMUs

<table>
<thead>
<tr>
<th>PMU number</th>
<th>Generators with PMUs</th>
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</thead>
<tbody>
<tr>
<td>14</td>
<td>1,8,9,10,15,16,17,18,19,20,21,22,24,27</td>
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<td>15</td>
<td>1,4,8,9,10,15,16,17,18,19,20,21,22,24,27</td>
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Rotor Angle Stability Analysis Results

Line Trip

MLE = -0.2355 \( \leftrightarrow \) System is stable

Generator Trip

MLE = 0.72849 \( \leftrightarrow \) System is unstable

Conclusions

- Developed MLE and State Calculator as effective techniques to predict loss of synchronism in power systems.
- Developed an index to quantify the level of observability for different PMU deployment scenarios.