Least costly probing signal design for power system mode estimation

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Outline

• Motivation
• Probing-based mode estimation
• Optimization problem formulation
• Results
• Conclusions
• Q&A
Oscillation monitoring

Oscillations if lightly damped can lead to a system black-out
Oscillations occupy transmission capacities and increase losses

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Frequency and damping ratio monitoring
Algorithms for measurement based mode estimation

- Transient response:
  - Prony, ERA, Pencil Matrix
  - Well established
  - Good accuracy
  - Not suitable for real-time monitoring

- Ambient and probing:
  - SysID and signal processing algorithms
  - Suitable for real-time monitoring
  - Lower accuracy
Probing based mode estimation

Power system
\[
dx/dt = Ax + Bu + Fz \\
y = Cx + Du + Gz
\]

Inputs
(load noise)

Outputs
(PMUs)

Deterministic signal

Probing signals
- FACTS devices
- AVR
- Turbine governors

Measured signals

Exactly known excitation brings new information that can be used for improved mode identification
Mathematical description of the probing-based mode estimation

- Model of the system

- In case of probing, the model is:
  \[ y(t) = \frac{B(z, \theta)}{A(z, \theta)} u(t) + \frac{C(z, \theta)}{D(z, \theta)} e(t) \]
  ✓ ARMAX
  ✓ Box Jenkins

- Mode estimation as an optimization problem

  \[ \min_{\theta} \frac{1}{N} \sum_{t=1}^{N} \varepsilon(t, \theta)^2 \quad \varepsilon(t, \theta) = y(t) - \hat{y}(t|t-1) \]
• The goal is to identify the critical damping ratio of $G(z)$
• The critical damping ratio is parameter of $G(z)$ (element of $\theta$)

$$P_{\theta}^{-1} = \left( \frac{N}{\sigma^2} \right) \frac{1}{2\pi} \int_{-\pi}^{\pi} F_u(\omega, \theta_0) F_u^{*}(\omega, \theta_0) \Phi_u(\omega) d\omega + \left( \frac{N}{2\pi} \right) \int_{-\pi}^{\pi} F_e(\omega, \theta_0) F_e^{*}(\omega, \theta_0) d\omega$$

**How should the probing signal look like??**

1) Length
2) Frequency spectrum
3) Time domain
Spectrum calculation

Requirements:
1) Control effort  
2) System disturbance  
3) Accuracy

Opt. criterion: \[
\min_{u(t)} J = \left( \frac{k_1}{2\pi} \int_{-\pi}^{\pi} \Phi_u(\omega) d\omega \right) + \left( \frac{k_2}{2\pi} \int_{-\pi}^{\pi} |G(s)|^2 \Phi_u(\omega) d\omega \right) \]

\text{Input} \quad \text{Output (frequency deviation)}

Constraint: \[
\text{var}(\zeta_i) = e_i^T P_{\theta} e_i < r \quad r - \text{tolerance}
\]

Keeping in mind:
\[
P_{\theta}^{-1} = \left( \frac{N}{\sigma^2} \right) \left( \frac{1}{2\pi} \int_{-\pi}^{\pi} F_u(\omega, \theta_0) F_u^*(\omega, \theta_0) \Phi_u(\omega) d\omega \right) + \left( \frac{N}{2\pi} \int_{-\pi}^{\pi} F_e(\omega, \theta_0) F_e^*(\omega, \theta_0) d\omega \right)
\]
Global algorithm (two steps)

- Spectrum calculation (LMI optimization)
- Time domain signal realization
  - FIR filter
  - Sample autocorrelation optimization
  - Multi-sine input signal [1]

Signal realization with constrained magnitude

- Power spectrum: \( \Phi_u(\omega) = \sum_{r=-m}^{m} c_r e^{i \omega r} \)
- Sample autocorrelation:
  \[ ACF_k(\tau) = \frac{1}{k} \sum_{i=\tau+1}^{k} u(i)u(i-\tau) \]
- Optimization:
  \[ \min_{u(k)} \sum_{\tau=0}^{M+K} \left( ACF_k(\tau) - ACF_{des}(\tau) \right)^2 \]
- Efficient recursive algorithm
  - Sample by sample
  - Every sample result of a simple optimization problem
Optimal probing signal design results

Minimized input

Minimized output

Input spectrum parameterization

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<th>White noise</th>
<th>Multi-sine</th>
<th>FIR filter</th>
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<td>var{y(t)}</td>
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<td>var{uy(t)}</td>
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Optimal probing allows us to reduce probing power and/or system disturbance while maintaining desired accuracy
Conclusions

• Monitoring of electromechanical modes is important
• Staged probing tests can provide a good accuracy
• Shape of the probing signal affects estimation accuracy
• Several considerations can be taken into account during design process
• Optimal probing allows us to reduce probing power and/or system disturbance while maintaining accuracy
• The proposed method is easy to implement
Thank you!

Questions?

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