

# Synchrophasor Based Robust Linear State Estimator

*Ali Abur, Murat Göl and Bilgehan Dönmez*  
*Department of Electrical and Computer Engineering*  
*Northeastern University, Boston*  
*abur@ece.neu.edu*



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# Measurement equations

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## *SCADA Measurements*

$$Z_S = h(X) + v \quad \text{Non-linear Model}$$

$$H_x : \nabla h(X) \quad \text{State} \quad \text{Measurement Error}$$

## *Phasor Measurements*

$$Z_P = H \cdot X + v \quad \text{Linear Model}$$

*H : Function of network parameters only*

A.G. Phadke, J.S. Thorp, and K.J. Karimi, "State Estimation with Phasor Measurements", IEEE Transactions on Power Systems, vol. 1, no.1, pp. 233-241, February 1986.

# Phasor-only Weighted Least Squares (WLS) state estimator

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$$Z = H \cdot X + v \quad \textit{Linear Model}$$

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*WLS state estimation problem:*

$$\textit{Minimize} \quad \sum_i^m \frac{r_i^2}{\sigma_i^2}$$

$$\textit{Subject to} \quad r = Z - H \cdot \hat{X} \quad \textit{residual}$$

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Non-robust: fails to provide unbiased estimates even when a single bad measurement exists!

# $L_1$ /Least Absolute Value (LAV) Estimator

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Minimizing the  $L_1$ -norm of residuals

$$\textit{Minimize} \sum_{i=1}^m |r_i|$$

$$\textit{Subject to} \quad Z = H \cdot \hat{X} + r$$

**Robust** up to a limited number of existing bad data.

# Pro/Cons of (LAV) $L_1$ estimator

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- Automatically rejects bad data given sufficient local redundancy, hence bad data processing is built-in.
- Higher computational load compared to WLS if there are no bad data.

# Definitions:

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- What are leverage measurements?

Estimated states are disproportionately sensitive to certain types of measurements due to their type and location. Such measurements will move the estimates towards wrong values when they carry bad data. They are called “leverage” measurements.

- What is scaling?

Measurement equations can be multiplied by appropriate constants to better “condition” the network matrices. Scaling can be applied in such a way to eliminate “leverage measurements” when the measurement equations are linear.

# Why was LAV not used before with SCADA measurements?

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- SCADA measurements
  - Nonlinear measurement equations
  - Iterative solution (higher cpu burden)

## **Why LAV is preferable now?**

- Major vulnerability of LAV is the so called “leverage measurements”. When PMU measurements are used it is possible to eliminate them by simple scaling.
- Recent advances in efficient Linear Programming (LP) code enable implementation for large scale systems.

# How does $L_1$ estimator tell which measurement is good which is bad?

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- $L_1$  estimator has the “interpolation” property, i.e. it fits the solution to a minimum number of required measurements for which the corresponding residual vector will have the minimum  $L_1$  norm.
- As a result, bad observation points (bad measurements) will automatically be left out during the solution.



# Properties of $L_1$ estimator

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- Efficient Linear Programming (LP) code exists to solve it for large scale systems.
- Use of simple scaling eliminates leverage points. This is possible due to the type of phasor measurements (either voltages or branch currents).
- $L_1$  estimator automatically rejects bad data given sufficient local redundancy, hence bad data processing is built-in.

# Simulation Results

3625 bus, 4836 branch utility system

	CPU Time (seconds)		
	No Bad Data	Single Bad Data	Five Bad Data
LAV with built-in BD removal	3.33	3.36	3.57
WLS using post-SE BD detection test	2.32	9.38	50.2

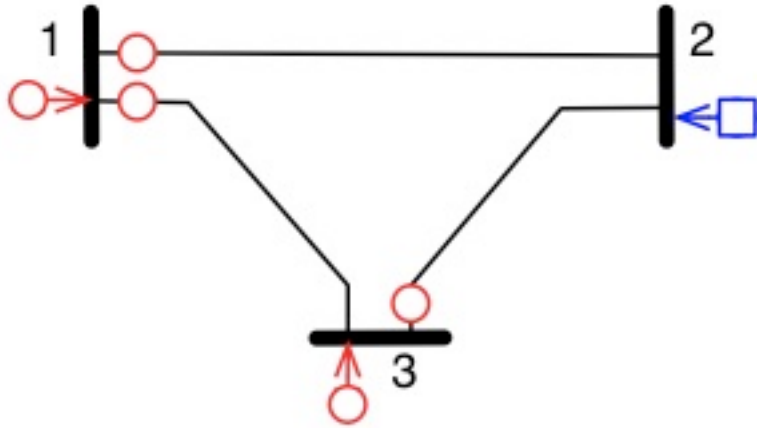
- Bad data handling of LAV solver remains fairly insensitive to the number of bad data.
- Bad data handling of WLS solver will be proportionally slower with increasing number of bad data in the measurement set.

# Remarks on CPU Results

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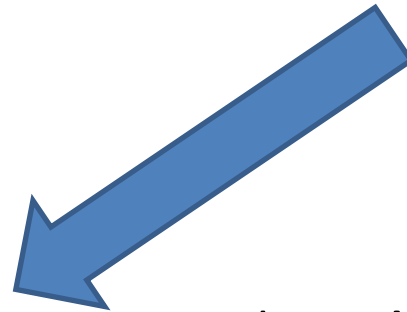
- WLS SE solution is faster when there are no bad data.
- Under bad data LAV SE performs comparable or better than WLS SE. Advantages become more pronounced with increasing bad data.
- WLS needs some form of post-SE bad data processor (e.g. largest normalized residual test) to detect and remove bad data.

# SCADA Based Implementation Options for SE



## Measurement set:

- Power injection measurements
- Power flow measurements
- Voltage magnitude measurements



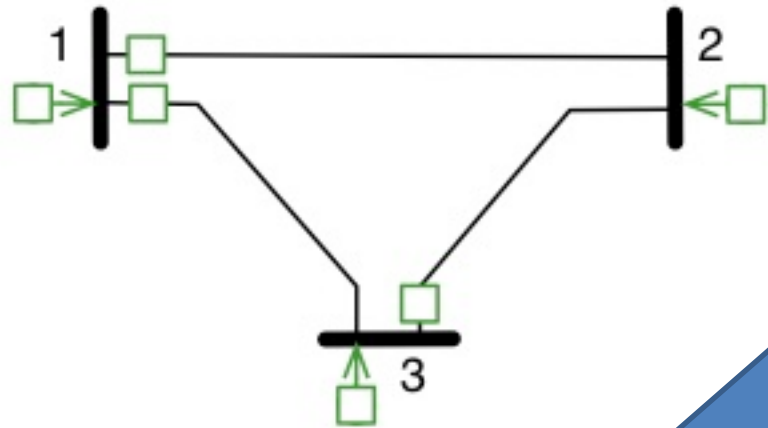
## Weighted Least Squares (WLS) Estimator

- Well-developed and widely-known
- Requires bad-data analysis (non-robust)
  - Normalized residuals test
  - Re-weighting

## Least Absolute Value (LAV) Estimator

- Linear programming based, computationally inefficient
- Does not require bad-data analysis
- Deficiency in the presence of leverage measurements

# PMU Based Implementation Options for SE



Phasor Measurement Units (PMUs):  
Linearly related states and  
measurements.

WLS :

- Linear solution
- Requires bad-data analysis
  - Normalized residuals test
  - Re-weighting (not applicable)
- **No deficiency** in the presence of leverage measurements, with *scaling*.

LAV (robust):

- Linear programming (**single step**), *computationally efficient*
- **Does not** require bad-data analysis
- **No deficiency** in the presence of leverage measurements, with *scaling*.

# Final Remarks

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- SE performance have significant impact on all other application functions related to the transmission grid
- LAV based Phasor-Only SE has superior performance in the following areas:
  - Computational speed
  - Robustness against bad / missing data

# Thank You Any Questions?



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