Metrics for Determining the Impact of Phasor Measurements on Power System State Estimation

Eastern Interconnection Phasor Project

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Table of Contents

Table of Contents

List of Exhibits:

List of Tables:

1. Introduction

The installation of time synchronized power system measurement devices to improve the reliability and security of the electric grid has increased steadily in North America over the last decade. This category of device includes phasor measurement units (PMUs). To promote the implementation of PMUs, the Department of Energy has established the Eastern Interconnection Phasor Project (EIPP). The EIPP seeks to promote PMU installation along with the planning and implementation of information systems and applications that will maximize the benefits of phasor measurements.

1.1 Project Objective

One of the applications that many feel will benefit from the addition of phasor measurements is the state estimator[1]. Because the State Estimation function is generally recognized to be critical for reliable grid and efficient market operation, the EIPP wishes to understand better the specific impact that the addition of phasor measurements has upon state estimator performance. Toward this end, the EIPP has engaged KEMA to investigate the impact of phasor measurements on the performance of the state estimator.

The project has two objectives: first, to determine the state estimator metrics that can be used to measure performance, and second to propose a process for testing the defined state estimator metrics.

1.2 Study Process

The study consisted of the following tasks:

1. A search of published work regarding phasor measurements and the state estimator to establish the foundation for the study.

2. Using the results of the literature search and KEMA's experience with state estimator procurement and implementation, identify a set of performance metrics.

3. Have state estimator vendors along with academic and industry experts review and comments on the metric set and identify and improve those metrics that can be practically tested for phasor measurement impact.

4. Develop the process for testing the impact of phasor measurements on the metrics of the state estimator.

[1] The final version of this paper will have clarification indicating that we are looking at an extension of the commercial state estimator programs commonly found embedded with the EMS in a control center environment. Further, a bit more explanation of what is meant by 'phasor measurement' will be given.

5. Have the stakeholders review the test process and incorporate their comments into the study.

6. Prepare a study report.

1.3 Report Outline

This report was prepared to describe:

- The set of metrics that can be tested to demonstrate the impact of incorporating phasor measurements as input to the state estimator

- The test methodology for measuring the impact of phasor measurements on state estimator metrics.

The report consists of the following sections:

1. *Introduction*

   This section describes the objectives of the study, the study methodology, and the report contents.

2. *State Estimator Metrics*

   This section identifies and describes the metrics that can be used to measure the impact of phasor measurements on the state estimator.

3. *Test Methodology*

   This section describes the test methodology to verify the impact of phasor measurements on state estimator results using the metrics identified in Section 2.

4. *Recommendations*

   This section includes recommendations resulting from the conclusions of the study.

2. State Estimation Metrics

It is desired that a commercial state estimator excel in four principal areas:

- **Accuracy** – The state estimator's principal function is to determine actual system conditions using noisy measurements. It is desired that estimated quantities be as close as possible to their true values.

- **Performance** – The outputs from the state estimator are needed in a timely fashion by other applications in a control center environment.

- **Robustness** – In practice, commercial state estimators must operate in the presence of measurement errors that violate uncertainty assumptions, network topology that differs from that reported by the SCADA system, and network model parameters that are inconsistent with observed power flow measurements. A good state estimator is robust in that it is able to detect and correct for these problems.

- **Completeness** – The state estimator is required to provide estimates of power system quantities for as much of the network as possible.

The sections that follow propose metrics for each of these categories. The purpose of these metrics will be to provide a basis for measuring the effects phasor measurement units will have on the results of commercial state estimators. This places the requirement on any proposed metric that it should not make any assumptions about or place any restrictions on the algorithms used to perform the estimation. While the literature is replete with comparisons of different estimation algorithms, commercial state estimators wrap proprietary algorithms, and metrics of their performance thus must necessarily take a 'black box' perspective. In addition, a good metric should be relatively easy to design and test for. Difficult tests are less likely to be executed.

With all these criteria in mind, an initial set of metrics was identified and developed by KEMA. The set was reviewed, augmented, refined and culled by a panel of helpful reviewers[2]. The final recommended set of metrics and supporting comments owe much to their contributions.

Because nearly every control area or electricity market is connected to neighboring transmission systems that influence system behavior, it is nearly always necessary to extend the network model used for state estimation beyond the primary regions of interest into areas where modeling is typically less precise and

[2] Special thanks go to Ali Abur, John Allemong, and the state estimation and PMU experts at ABB, AREVA and Siemens. Their suggestions have been invaluable in determining metrics that are both useful and reasonable to apply to modern commercial state estimators.

telemetry less pervasive. Accordingly, it is expected and accepted that estimation of power system quantities will be better for the immediate network than for outlying regions. In this document, these networks shall be referred to as the internal and external networks, respectively.

The terms internal and external are intentionally loosely defined, as an appropriate partition of the network model into these two components will vary by state estimator application. The metrics introduced in the remainder of this document will refer often to the internal network exclusively, with the assumption that the internal network corresponds to the regions where state estimator performance is needed to be the best. This is not a prescriptive limitation, however, and all metrics identified within this report can be applied to the internal, external, or whole networks as needed.

2.1 Accuracy

The intended use of a state estimator solution must be considered when defining the criteria for accuracy metrics. In control centers and market operations centers today, the state estimator is required to deliver a power flow solution that, as closely as possible, represents current conditions in the field. As a tool for helping to ensure system security, it is important that the state estimator solution indicate to operators actual or near violations where they exist without triggering alarms from false positives. Further, to ensure confidence with the operators, it is important that for each bus the real and reactive power mismatch be very close to zero.

For market operations, the state estimator output is required to provide a power flow solution. This solution is commonly used to provide total system loss estimates, load bus allocations, and generator injections, all of which have direct effect on financial settlements with market participants. Further, the calculation of ex post locational clearing prices by some markets [Ott03] involves linearization around the quiescent operating condition provided by the state estimator solution.

2.1.1 Real-Time Accuracy Metrics

Unfortunately, it is very difficult to judge the accuracy of the state estimator during real time operation due to inability to know the actual system conditions. What can be ascertained in real-time, however, is whether the state estimator solution (including derived quantities) satisfies important necessary conditions to be a power flow solution.

The sum of both real and reactive power injections and flows into each bus must be identically zero to satisfy network power flow equations. For any given estimate of the state, the estimated real and reactive power flows through branches and shunts is uniquely (and easily) determined. The sum of these branch and shunt flows necessarily must be exactly offset by the estimated real and reactive net injections from generation and load.

Only for buses known to have no injection can a non-zero net injection estimate be known to be incorrect and labeled as a mismatch. Where the number of generators and loads is known to be exactly one (1) at a bus, the estimated injection for that apparatus may be uniquely assigned. In this case, it is not proper to consider the difference between the estimated injection and available local injection telemetry as a mismatch indicating inaccurate estimation; the difference is only a measurement residual the statistics of which can be examined in the context of bad data identification.

When multiple devices exist, network equations do not provide a mechanism to determine the split of the estimated net injection among the devices; if individual injection estimates are required (as they are for real time market clearing, for example) then a heuristic allocation scheme must be applied. If the net injection to be assigned among the devices at the bus is consistent with the known capabilities of the devices, a full allocation of the injection should be straightforward. In other cases, (e.g. allocation of a net injection of 300MW to two generating units, each known to be limited to 100MW in capacity), the algorithm designer will have to choose between device level consistency and nodal zero net injection.

With some reservation, the following are proposed as metrics related to injection mismatch:

$$M_{Pmismatch} = \max_{b \in Bint} \left| \sum_{j} P_j^b \right| \qquad (1)$$

$$M_{Qmismatch} = \max_{b \in B\mathrm{int}} \left| \sum Q_j^b \right| \qquad (2)$$

These metrics measure real and reactive power mismatch for the internal network, defined by the set of buses *Bint*. The quantities included in the sums for (1) and (2) are all the estimated real and reactive network flows and estimated injections at each bus. That is, the quantities used to calculate (1) and (2) should exactly match those corresponding to the 'power flow' solution from the state estimator used as input to subsequent processes and calculations. These metrics do not measure accuracy, only the ability of the estimator to meet an important necessary condition for a solved power flow solution.

2.1.2 Controlled Test Accuracy Metrics

Other metrics for state estimation accuracy can be applied by operating the state estimator in a controlled test environment where the 'true' system conditions are determined from a solved power flow and the 'actual' measurements are created by perturbing quantities derived from the power flow solution with noise. Such experiments are routinely performed by state estimator vendors during the refinement and tuning of their algorithms; a test bed for this purpose is described in Section 3. In short, the test procedure follows this pattern:

1) For a given network model, system conditions and topology, solve a power flow and extract voltage magnitudes, angles, branch real and reactive flows, injections, etc.;

2) Create measurement data to be submitted to the state estimator by perturbing the corresponding quantities from the power flow solution;

3) If desired, introduce topology or parameter 'errors' to alter the network model to be used by the state estimator;

4) Produce the state estimator solution, including derived quantities;

5) Evaluate the state estimator's performance by comparing quantities from the power flow solution to those derived from the state estimator solution.

The most obvious (and convenient) way to generate an accuracy metric is to choose a power flow solution quantity of interest (e.g. bus voltage magnitude) and define a norm-like calculation on the difference between the 'true' value (from the power flow solution) and the estimated value, derived from the state estimate.

Before discussing appropriate quantities for comparison it is worthwhile to examine some implications of the form of the metric. Consider, for example, a network model with n branches and three candidates to measure accuracy of real power flow Pf on these branches[3]:

$$\sum_{j=1}^{n} \left| Pf_j^{true} - Pf_j^{est} \right| \tag{3}$$

$$\sum_{j=1}^{n} \left(Pf_j^{true} - Pf_j^{est} \right)^2 \tag{4}$$

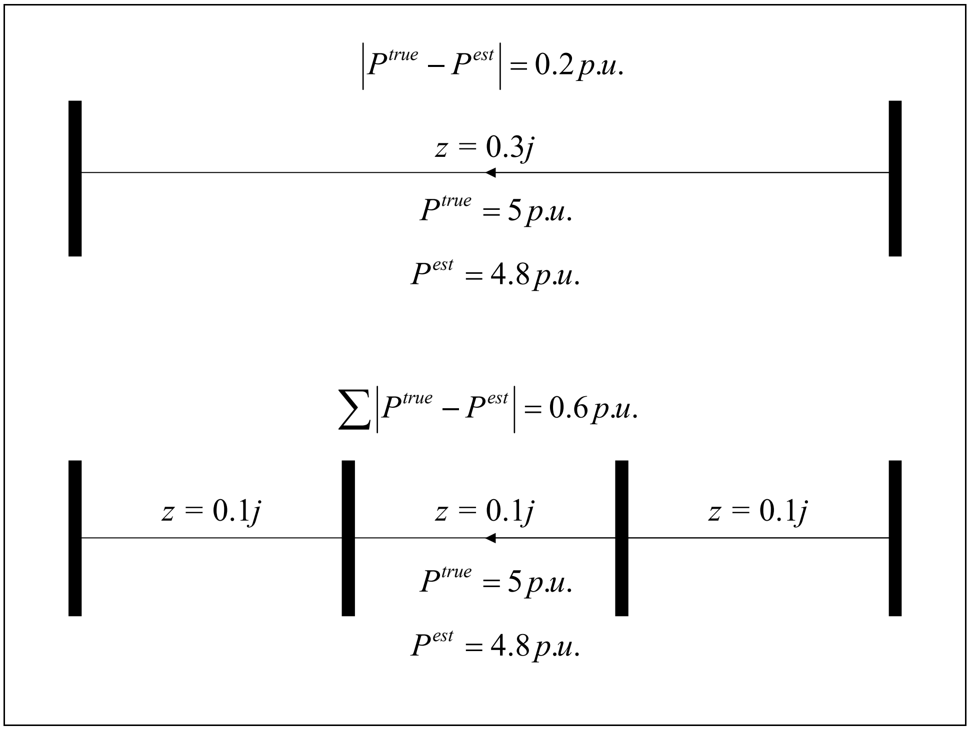
$$\max_{j=1\ldots n} \left| Pf_j^{true} - Pf_j^{est} \right| \tag{5}$$

The first of these, (3), is the 1-norm of the estimation error, and is proportional to the average error in branch flow estimation. The second, (4), is the square of the 2-norm (Euclidean norm) of the estimation error. The third, (5), is the infinity norm of the estimation error, i.e. the 'worst case' error. All three seem reasonable, in that larger values indicate worse performance, and zero error results in a zero calculation.

Each of (3) and (4) can return very different 'scores' for nearly equivalent network models with the same state estimator solution and same power flow solution. The top half of Exhibit 2-1 shows a simple

[3] Here, and elsewhere in this section the superscripts *true* and *est* denote 'true' and 'estimated' quantities, respectively. 'True' quantities are determined from the power flow solution, while estimated quantities are calculated from power flow equations evaluated using the estimated state.

Exhibit 2-1: Effect of Line Segments on Flow Metrics



(lossless) branch with a 0.2 p.u. error in estimation of real power flow. In the bottom half of the diagram, additional buses have been added to split the line into three line segments; the actual and estimated flows remain the same. With three line segments instead of one, the contribution to the 1-norm (3) is three times as much, although both models represent equivalent subsets of the network.

Only the infinity norm, (5), is immune to these effects, in that it always reports the 'worst case' estimation error. Unlike the first two, however, one lone bad estimation error can result in a poor score, incorrectly suggesting the estimation was poor.

In conclusion, it is perhaps important to keep in mind that metrics similar to those shown above usually lack a meaningful physical interpretation, and are somewhat sensitive to model construction. Accordingly, while they are useful to compare differences in algorithm or differences in measurements for a fixed network model, their use in other circumstances should be approached with caution.

With these caveats in mind, two simple metrics are now introduced that judge the state estimator's ability to estimate the most important quantities defining a power flow solution: voltages and flows.

An important attribute of the state estimator is its ability to predict real and reactive power flow on branches. By combining these as one complex quantity ($\vec{S} = P + jQ$), the following metric for complex power flow estimation accuracy is proposed:

$$Macc_{Sflow} = \left(\sum_j \frac{\left| \vec{S}^{true}_{j,\text{from}} - \vec{S}^{est}_{j,\text{from}} \right|^2 + \left| \vec{S}^{true}_{j,\text{to}} - \vec{S}^{est}_{j,\text{to}} \right|^2}{MVA_j^2} \right)^{\frac{1}{2}} \tag{6}$$

In (6), the summation index j ranges over all branches in the internal network; as indicated in the equation, error terms for complex flow at each end of the branch should be included. The term MVA in the denominator is a limit of the branches capacity; its inclusion in recognition that a 10MW estimation error, for example, is more operationally significant for a small 69kV line than for a large 500 kV branch. Importantly, note that each squared term is the magnitude of a difference of complex numbers, not the difference of magnitudes.

A commonly reported accuracy metric for state estimation in many fields of application, including power system state estimation, is a norm of the error of the state estimate. For nearly all power system state estimator implementations, nearly all the elements of the state vector consist of bus voltage magnitudes and angles. While it is reasonable to compare voltage magnitude errors and voltage angle errors separately, it is straightforward to combine both in a norm metric that captures the effect of both:

$$Macc_V = \left\| \vec{V}^{error} \right\|_2 = \left(\sum_j \left| \vec{V}^{true}_j - \vec{V}^{est}_j \right|^2 \right)^{\frac{1}{2}} \tag{7}$$

In (7), $\vec{V}^{true}_i$ and $\vec{V}^{est}_i$ are the 'true' and estimated complex phasor voltage at the jth bus of the internal network reported on a per unit basis. It is important, of course, that the same reference bus be used for both 'true' and estimated voltages.

The accuracy metrics (6) and (7) were designed and chosen to test the ability of the state estimator to match a controlled test model in the aggregate; it is suggested that a state estimator that returns good (low) values on these metrics and on the mismatch metrics (1) and (2) has done a good job of producing a power flow solution matching system conditions. The use of the 2-norm is a compromise between the 1-norm, related to the average estimation error over estimated quantities, and the infinity norm, which while useful for 'worst case' analysis punishes state estimators which produce only a handful of bad estimates. The 2-norm is comprehensive in that it sums over all quantities, and punishes bad estimates with the quadratic term.

2.1.3 Other Candidate Measures of Accuracy

The value of the objective function is nearly always readily available upon completion of state estimation. Certainly for fixed inputs (measurements, topology and model parameters), an improvement in the objective function indicates superior state estimator performance. It is tempting to compare objective function values with and without phasor measurements to measure the impact of the phasor measurements on the state estimator performance.

Consider the optimal objective function value J_1 for a weighted least square (WLS) implementation with m (non-PMU) measurements z_i, *where* $i = 1 \ldots m$:

$$
\begin{aligned}
J_1 &= \min_{\hat{x}} \sum_{i=1}^{m} \left(\frac{z_i - h_i(\hat{x})}{\sigma_i} \right)^2 \\
s.t. \quad & c(\hat{x}) = 0
\end{aligned}
\tag{8}
$$

In (8), the vector $\hat{x}$ is the estimate of the state, $h_i(\hat{x})$ are the nonlinear power flow equations describing the m measurements, and $c(\hat{x}) = 0$ are (optional) constraints representing, for example, equality constraints included for zero-injection buses. Adding only p new measurements (from PMUs or elsewhere) and re-estimating always yields an objective function value J_2 that is at least as great as J_1:

$$
\begin{aligned}
J_2 &= \min_{\hat{x}} \left(\sum_{i=1}^{m} \left(\frac{z_i - h_i(\hat{x})}{\sigma_i} \right)^2 + \sum_{j=1}^{p} \left(\frac{z_j - h_j(\hat{x})}{\sigma_j} \right)^2 \right) \\
&\geq \min_{u} \sum_{i=1}^{m} \left(\frac{z_i - h_i(u)}{\sigma_i} \right)^2 + \min_{v} \sum_{j=1}^{p} \left(\frac{z_j - h_j(v)}{\sigma_j} \right)^2 \\
&= J_1 + \min_{v} \sum_{j=1}^{p} \left(\frac{z_j - h_j(v)}{\sigma_j} \right)^2 \\
&\geq J_1
\end{aligned}
\tag{9}
$$

The inequality shown in (9) is derived as follows: The first line (equality) is the minimum over set $\hat{x}$ of the combined sum and is the value of the objective function when all $m + p$ measurements are considered. The second line states that this minimum is greater than or equal to the sum of the minima over the two sets of measurements (the sets u and v are of the same dimension as $\hat{x}$). The third line recognizes the first minimum in line two as exactly J_1 from (8) above. The last line recognizes the second minimum to be nonnegative and draws the inequality relationship.

Accordingly, the objective function value cannot be used as a metric for accuracy to compare state estimator performance with different measurement sets. It is straightforward to show that other commonly used objective functions also must increase with added measurements. For this reason, despite its ease of application, the objective function should not be used as a metric when comparing the impact of new or different measurements (e.g. from PMUs) on the state estimator solution.

An accuracy metric for controlled tests cited in the literature is the Error Estimation Index (EEI) [Slu95], given as

$$EEI = \sum_{i=1}^{m} \left(\frac{z_i^{true} - z_i^{est}}{\sigma_i} \right)^2 \tag{10}$$

where σ_i is the actual standard deviation of the Guassian, zero-mean random noise used to pollute the noise-free measurement z_i^{true} in creating the noisy measurement z_i . The introduction of the normalizing factor σ_i in the denominator effectively serves to discount measurement estimation errors for noisy measurements. Recalling that the stated objective of the accuracy metric is to measure the ability to produce an accurate power flow solution, the EEI suffers in comparison to the metrics recommended above for two reasons. First, the introduction of the term σ_i eliminates from the calculation any power system quantities that are not in the measurement set. While improving measurement accuracy is one goal of a state estimator, as important is the ability to provide accurate visibility of unmetered quantities. Second, the importance of an accurately estimated quantity to the end user of the state estimator bears no relation to the measurement error variance; the normalizing factor σ_i is not appropriate in this context.

A similar candidate metric is the performance index (PIP) used in [Živ96] to determine the ability of the state estimator to accurately discern real power flow measurements:

$$PIP = \frac{\sum_{i=1}^{m_{pf}} \left(Pf_i^{true} - Pf_i^{est} \right)^2}{\sum_{i=1}^{m_{pf}} \left(Pf_i^{true} - Pf_{ei}^{meas} \right)^2} \tag{11}$$

where Pf denotes real power flow on m_{pf} branches. For good estimation, the estimate of each flow will lie closer to the true than will the measured value, and the entire metric will be less than one. The introduction of the actual flow measurements in the denominator, however, eliminates the possibility of including estimation accuracy for unmetered lines. For this reason, (11) is not recommended as an alternative to (6).

2.2 Performance

While it is desirable that the state estimator program be as accurate as possible, there are additional performance requirements observed in practice that relate to the ability of the state estimator to deliver to downstream processes a stable solution in reasonable and predictable time.

2.2.1 Convergence

The precision, if not the accuracy, of a state estimator that uses an iterative solution scheme can be judged by its ability to converge. The confidence that users of real-time state estimators have in the value of the solution is diminished if significant changes in state variables or objective function are still occurring at the terminal iteration. The following three metrics are either known to be in use today in control center environments or are minor variations thereof:

$$Mconv_{obj} = \left| 1 - \frac{J^{k_{term}}}{J^{k_{term}-1}} \right| \tag{12}$$

$$Mconv_V = \max_{i \in B_{int}} \left| 1 - \frac{V_i^{k_{term}}}{V_i^{k_{term}-1}} \right| \tag{13}$$

$$Mconv_\theta = \max_{i \in B_{int}} \left| \theta_i^{k_{term}} - \theta_i^{k_{term}-1} \right| \tag{14}$$

For each of these, k_{term} denotes the terminal iteration. The metric $Mconv_{obj}$ shown in (12) measures the relative change in objective function value J at the last iteration, while the metric $Mconv_V$ shown in (13) measures the largest final relative change in bus voltage magnitude over the buses of interest. Note that $Mconv_\theta$ uses the absolute difference to avoid problems when the angle is near zero, which will occur near the system reference bus.

A clarifying example of use of these metrics is given by the following statement: 'The real-time state estimator solution shall be deemed acceptable if at termination the power mismatch metric $M_{Pmismatch}$ is no larger than 50 MW and both of the convergence metrics $Mconv_V$ and $Mconv_\theta$ are no larger than 0.002'.[4] This statement very closely matches the criteria used by a large North American ISO to monitor their real-time state estimator performance.

[4] This example is a rewording (using metrics defined in this document) of the condition the Midwest ISO published as the acceptable criteria for the estimator prior to market start. Found in a document simple titled 'Metric Interpretive Guidance 03 Completed and Verified' at http://www.midwestmarket.org/publish/Document/2df21_101f7800020_-7f840a48324a?rev=1 (link valid as of 09 Jan 2006), the original quote is 'State Estimator Solution must converge to .002 per unit voltage magnitude and/or angle and maximum power mismatch of 50MW State Estimator model within the Midwest ISO Market footprint.'

Another convergence metric is suggested by a necessary condition for the state estimator to find the optimum solution, namely $\frac{\partial J}{\partial \hat{x}} = 0$. If a conventional weight least squares algorithm is used, the objective function is as follows:

$$J(\hat{x}) = \sum_{j=1}^{m} \left(\frac{z_j - h_j(\hat{x})}{\sigma_j} \right)^2 \tag{15}$$

and the necessary condition for optimality is given by the vector equation

$$g(\hat{x}) = \frac{\partial J}{\partial \hat{x}} = -2\sum_{j=1}^{m} \left(\frac{z_j - h_j(\hat{x})}{\sigma_j^2} \right) \frac{\partial h_j(\hat{x})}{\partial \hat{x}} = 0 \tag{16}$$

Accordingly, it may be of interest to report $\left\| g(\hat{x}^{k_{term}}) \right\|_2$ as a scalar measure of the satisfaction of (16) at the terminating iteration k_{term}. However, the burden introduced by the complexity of the calculation does not warrant the recommendation of this measure and a convergence metric for general use. Algorithm designers and researchers may find merit in its use, however.

2.2.2 Timing

The faster that state estimation software executes successfully, the more realistic a picture of evolving system conditions the operators will have. Ideally, the estimated state would be available as soon and as often as the telemetry is updated. A candidate timing metric is the average execution time in seconds for state estimation, taken over a significant sample of cases.

Measuring process timing precisely and consistently on real-time systems operating on computers with many shared processes, however, can be difficult. Recall that the motivation for this study is the need to measure the benefits that inclusion of phasor measurement units have on state estimation. It is the majority opinion of the contributors to this study that any effect on execution time from the augmentation of the measurement set with phasor measurements will be not meaningful. Accordingly, and because in nearly all control center environments algorithmic execution time is not perceived to be a limiting factor on system performance, no timing metric is recommended for broad use at this time.

2.3 Robustness

Robustness is the ability of the state estimator algorithm to perform its function in other than ideal conditions. In particular, commercial state estimators are required to perform in the presence of bad measurement data, bad network topology information, and bad or uncertain parameters used to describe the power system network model. In addition, it is of interest to know the sensitivity of overall estimation accuracy to the accuracy of the measurement system. Missing measurements are addressed in a subsequent section on completeness.

2.3.1 Bad Measurement Identification and Detection

An important property of any state estimator in use is the ability to identify and reject bad measurements, *i.e.* measurements for which the observed data has error that is inconsistent with the modeling assumptions. While bad measurement (and topology, and parameter) rejection will impact a state estimator's ability to score well on accuracy tests with properly designed experiments, it is also a simple matter to construct tests to measure bad measurement identification explicitly. A flexible metric for this purpose is given as follows:

$Mmeas_{\sigma}(p, a) =$ % of cases the state estimator correctly identifies all p bad measurements created by adding a constant term of $a\sigma$ or $-a\sigma$ (with equal likelihood) to the measurement, where σ is the assumed standard deviation of the measurement error. (17)

Most commercial state estimators are easily capable of flagging single or isolated bad measurements; accordingly, the parameter p in (17) allows the test designer to build more difficult test cases where multiple measurements are bad. Multiple sets of input data should be created to use this metric, and the same data sets should be used when comparing algorithms or comparing measurement sets. Finally, it is recommended that the parameter a in (17) be greater than 1.5, so as not to generate measurements that could alternately be viewed as either bad or good but unlikely.

The description of (17) requires the tester to measure the algorithm's ability to identify 'correctly' all p bad measurements. Two acceptable standards for correctness are:

- Explicit identification of exactly the p bad measurements;
- Identification of the p bad measurements, but also with the identification of other measurements as bad (false positives).

The former of these is obviously preferred to the latter, but even the latter may be considered acceptable; a state estimator that 'accidentally' discards a few good measurements is usually still able to produce a very good state estimate. When using (17) the tester should be explicit as to what defines correct bad measurement detection when reporting results.

Other candidate measures for bad data detection are:

- % of cases the state estimator correctly identifies all p bad measurements created by setting the measurement value to zero;

- % of cases the state estimator correctly identifies all p bad measurements created by setting the measurement values to high limits;
- % of cases the state estimator correctly identifies all p bad measurements created by setting the measurement values to low limits.

It was judged that each of these was sufficiently similar to (17) that their recommendation as metrics was not warranted.

Bad meter calibration can be simulated by perturbing measured values with an affine or quadratic transformation. Typically, multiple measurement scans are required to detect non-random calibration errors using a generalization of state estimation known as remote meter calibration. While the use case of meter calibration via generalized state estimation is beyond the scope considered here, it is a straightforward exercise to create metrics to measure performance.

It is worthwhile considering a metric for the detection of bad data (necessary but not sufficient for bad data identification). Consistent with the assumptions required for WLS estimation to be maximum likelihood estimation, the measurement errors are usually assumed to be Guassian with zero mean and known variance. This means that for data that are not 'bad' that each measurement error when normalized by dividing by its standard deviation will have a standard normal distribution

$$e_i^N = \frac{e_i}{\sigma_i} \sim N(0,1) \tag{18}$$

The sum of squares of m independent standard normal variables is a random variable with a χ_m^2 distribution having mean m and variance $2m$. If these same variables are coupled by n constraints, their sum of squares with m-n degrees of freedom has a χ_{m-n}^2 distribution with mean m-n and variance 2(m-n). It follows that the sum of normalized measurement errors follows such a distribution:

$$J(\hat{x}) = \sum_{i=1}^{m} \left(\frac{z_i - h_i(\hat{x})}{\sigma_i} \right)^2 \sim \quad \chi_{m-n}^2 \tag{19}$$

Clearly large normalized measurement errors will make the sum $J(\hat{x})$ large. A single instance of this sum available after state estimation can be compared to the probability pdf for the χ_{m-n}^2 distribution to determine the probability p the a random draw of the distribution can be as large as the observed sum:

$$p = \Pr\left(\chi_{m-n}^2 > J(\hat{x})\right) \tag{20}$$

Many commercial state estimator programs compare p to a confidence threshold to decide whether the measurement set contains bad data. If the threshold remains fixed, then p can be used as a metric to forecast whether the state estimator algorithm can detect (if not identify) bad data. Because the inclusion of such calculations is non-trivial, and because most commercial vendors will already be using χ^2 analysis in the determination of (17), the application of the technique in a metric is judged not warranted.

2.3.2 Bad Topology Detection

Perhaps the greatest obstacle to reliable and accurate power system state estimation is the not uncommon occurrence of the actual network topology differing from the assumed network model because of inaccurate knowledge of switch settings. When compared to bad measurement detection, for which there are reasonably straightforward and commonly accepted hypothesis tests and strategies for bad measurement identification and elimination, network topology identification algorithms are computationally more difficult, more recent in design, and therefore less pervasive in commercial application.

Very often the SCADA system relies upon manual operator entry of switch settings to define branch connectivity, and these data are prone to inaccuracy. Operating with an incorrect topology, the state estimator struggles to find a system state vector consistent with measured values. Accordingly, recent advances in commercial state estimation algorithms known alternately as generalized state estimation or topology estimation have been focused on the determination (and correction) of incorrect open/closed breaker settings. Topology estimation can be embedded within the state estimation algorithm or packaged as a distinct process.

The suggested metric is similar to those proposed for bad measurement detection:

$$Mtop(p) = \text{\% of cases the state estimator correctly identifies all } p \text{ topology errors.} \qquad (21)$$

It should be noted that in actual real-time use, incorrect switch settings are often coincident with gross (analog) measurement errors, making more difficult the accurate determination of both. The enterprising tester may consider devising test cases to measure both (17) and (21) together.

2.3.3 Parameter Estimation

The power system model also may have substantially incorrect modeling parameters for branch reactance, resistance, and line charging susceptance that adversely impacts the state estimator's ability to determine an accurate state estimate. Some commercial state estimators today are capable of parameter estimation, where questionable model parameters are treated as augmentations of the state vector so that they may be estimated. If the software permits the detection and estimation of questionable values, parameter 'errors' can be introduced into the test case and a metric that is similar to (17) may be used.

2.4 Completeness

In addition to refining the accuracy of measurements, the state estimator is required to provide reasonable estimates of measurable power system quantities where measurement devices do not exist. The extent to which this is possible at any given time is measured by observability; the robustness of observability with respect to measurement loss is redundancy.

2.4.1 Observability

One of the primary goals of state estimation is to provide the estimation of measurements on branches and at buses where no telemetry exists. It is a standard requirement for commercial state estimation software to determine the unobservable areas of the network and to generate a minimal set of pseudomeasurements that restores system observability.

Much published work exists (e.g. [Bei05], [Nuq01]) on cost-effective methods to increase system observability by PMUs, using techniques very much the same as those used for optimal meter placement. Most optimal placement techniques attempt to restore to full rank matrices associated with either the topology or the gain matrix[5]. Accordingly, it is tempting to propose metrics for observability based upon the rank of various matrices. However, doing so tends to impose an algorithmic approach upon the estimation algorithm, which is not appropriate for the purpose of this study.

Instead, it is proposed to let the estimation algorithm use its own methods for determining observability and report the number of in-service buses that are unobservable and require pseudomeasurements:

$$Mobs = \text{ \# of in-service buses of interest that are unobservable} \qquad (22)$$

The phrase 'of interest' in (22) allows the metric to be applied in a manner consistent with use of the state estimator (e.g. the internal network). While identification of the unobservable portion of the network is nontrivial[6], this functionality is considered standard on modern commercial state estimation programs. Accordingly, (22) is recommended because its application in a testing environment is expected to be straightforward.

[5] The gain matrix, usually denoted by G in the state estimation literature, defines the relationship between the gradient of the objective function and updates to the state estimate for iterative algorithms. If weighted least squares estimation is employed with higher order terms ignored, G takes the state ($\hat{x}$) dependent form $G(\hat{x}) = H(\hat{x})^T WH(\hat{x})$ where H is the gradient of the power flow equations and W is a diagonal weighting matrix.

[6] The bibliographic references in the next section (Redundancy) provide a good overview of applicable techniques for determining observability.

2.4.2 Redundancy

Redundancy[7] relates to the effect upon estimation of the loss of one or more measurements. The most important impact of loss of measurements is the loss of system observability; a measurement that is required to maintain observability is said to be *critical*. Further, each measurement belongs to exactly one *critical set*, defined as a set of measurements the loss of all of which is required to reduce observability. Clearly every critical measurement is the only member of its critical set. For each measurement, one can define a *measurement redundancy index* equal to the cardinality (size) of its critical set minus one. The greater a measurement's redundancy index, the easier it is for the state estimator to detect and correct for gross errors in its measured values.

Huang and Lei [Hua01] propose a system redundancy index to summarize the redundancy provided by a set of measurements. If p_i is the redundancy index of measurement i, $i = 1...m$, the system redundancy index P is given as the average measurement redundancy index:

$$P = (\sum_{i=1}^{m} p_i)/m \tag{23}$$

For a critical measurement, the measurement residual defined as the difference between the measurement value and the estimated measurement value $r_i = z_i - h_i(\hat{x})$ is zero. That is, any error in the measurement is transferred directly to the estimated measurement. For any given measurement, a useful measure of redundancy is the ratio of the variance of the measurement residual to the variance of the measurement error. As redundancy improves, the sensitivity of the estimated measurement to measurement errors will decrease, and the ratio of the variance of the residual variance to the measurement error variance will approach unity. A suggested measure of redundancy for the entire system is the minimum ratio over all measurements:

$$\min_{j=1...m} \frac{\rho_j^2}{\sigma_j^2} \tag{24}$$

Where ρ_j^2 and σ_j^2 are the residual variance and measurement error variance of the jth measurement, respectively. If weighted least squares estimation is used with the error variances used for weighting (in the usual manner), then the quantity ρ_j can be found by the relationship

[7] The definitions in this paragraph are a synopsis of related material in [Hua01]].

$$\rho_j^2 = \sigma_j^2 - \text{diag}_{jj}\left(H\left(H^t R_z^{-1} H\right)^{-1} H^t\right) \tag{25}$$

where H is the (state dependent) Jacobian of the power flow equations and R_z is the covariance matrix of the measurement errors.

Each of the proposed and similar measures of redundancy is algorithm dependent and difficult to implement in a testing environment. Accordingly, no redundancy metric is recommended at this time. Redundancy measures can play an important role in sensor placement algorithms (see for example [Hua01], [Mon99] Ch. 7, [Abu04] Ch. 4), about which a bit more will be said in Section 2.5.

2.5 PMU Placement for State Estimation

Recall that the motivating factor behind the creation of the metrics summarized in this section is to define a framework for evaluating the impact of phasor measurements on state estimator results. In general, most utilities and control areas today have few (if any) PMUs in place, and also have telemetry in place to provide observability for the internal network. Accordingly, when considering the location of new PMUs to increase state estimator performance in a cost effective manner, the problem to be solved is one of incremental measurement placement – finding the best places to add 'meters' to a measurement system that is already fairly well metered.

For the incremental measurement placement problem, it is likely that accuracy in the state estimator solution is deemed the metric for which improvement is wanted most. For systems with very few meters, improving observability may be the most important criteria. Considerable effort has been devoted to meter placement to improve observability (see for example [Mon85], [Cle90]) resulting in techniques applicable to sensors of all types, including PMUs. More recently, researchers have devoted specific attention to algorithms for improving observability using PMUs in particular (see for example [Xu04], [Bei05] and [Nuq01]).

Most algorithms for improving accuracy of the state estimator solution via incremental meter placement do so by attempting to reduce uncertainty in the estimate of the state vector; [Çel95] is a good example. The inverse of the gain matrix, $G^{-1} = \left(H^T R_z^{-1} H\right)^{-1}$ is easily shown to be the covariance matrix of the state error vector $x - \hat{x}$ when weighted least squares estimation is used, and the Jacobian H is evaluated at the final solution. Quite sensibly, many published algorithms identify candidate measurements for addition by looking for ways to reduce the large diagonal elements of G^{-1} (computationally efficient using sparse inverse techniques). In [Çel95], for example, a subset of available measurements are identified that are proximal to buses corresponding to large state error variances; from each measurement in that subset the diagonal elements of G^{-1} are reevaluated with the measurement added and the ones with the greatest variance reductions are ultimately selected.

In other fields where state estimation is regularly performed (especially target localization and tracking via image sensors (good recent examples are [Den02], [Xio02])) the information theoretic concept of entropy is used identify optimal sensor placement. First introduced by Shannon ([Sha48][8]), entropy (also known as information entropy, Shannon entropy, and differential entropy) is a scalar measure of the uncertainty associate with a multivariate random distribution. For the estimated state vector $\hat{x}$, the Shannon entropy is given by the integral involving the multivariate density function $f(\hat{x})$ as

$$\begin{aligned} H(\hat{x}) &= -\int_{-\infty}^{\infty} f(\hat{x}) \ln(f(\hat{x}))dx \\ &= -\int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} f(\hat{x}_1, \hat{x}_2, \ldots, \hat{x}_n) \ln(f(\hat{x}_1, \hat{x}_2, \ldots, \hat{x}_n) d\hat{x}_1 d\hat{x}_2 \ldots d\hat{x}_n \end{aligned} \tag{26}$$

An algorithm based on reduction in entropy to site new meters for state estimation can be found in [Phu77], the key concepts of which are now summarized here. If all the 'usual' assumptions regarding Guassian measurement errors are made and weighted least squares estimation is assumed, then it is straightforward to show that the Shannon entropy is given by

$$H(\hat{x}) = ½n[1 + \ln(2\pi)] + ½ \ln\left|\det\left(G^{-1}\right)\right| \tag{27}$$

where n is the number of states. If $LU = G$ is a decomposition of G into lower (L) and upper (U) triangular square matrices, then the last term in (27) can be simplified:

$$\ln\left|\det(G^{-1})\right| = \ln \prod_{i=1}^{n} |U_{ii}| = -\sum_{i=1}^{n} \ln|U_{ii}| \tag{28}$$

Determining the reduction in entropy associated with adding a new measurement requires only to update the diagonal elements of the matrix U resulting from the change to G and calculating the sum shown in (28).

An example shows how entropy might be used to site PMUs to improve measurement accuracy. Consider the IEEE 300 bus test system[9], which is sufficiently large enough to be nontrivial and small enough to facilitate algorithmic investigation. A base measurement set was created for this system as follows:

[8] A good complement to this paper for those interested in the history of entropy and estimation is [Wei70].

[9] This system, originally assembled by a IEEE Power Engineering Society Test Systems task force in 1993 under the direction of Mike Adibi, is currently available for download from the University of Washington Power System Test Case Archive maintained by Richard Christie at http://www.ee.washington.edu/research/pstca/.

- Real and reactive power flow measurements were added at one end of each of the 411 lines. For each measurement, a standard deviation of 1.39%[10] of full scale rating was used, where full scale rating was approximated as 150% of the MVA observed for the base case power flow.

- Real and reactive power injection measurements were added for each of 69 generating units. For each measurement, a standard deviation of 1.39% of full scale rating was used, where full scale rating was approximated as 150% of the injection MVA observed for the base case power flow.

- Voltage magnitude was measured at every 10% of out of 10 of 300 buses. For each measurement, a standard deviation of 0.89% of full scale rating was used, where full scale rating was approximated as 1.0 p.u.

The measurement scheme is, of course, purely fictional. The difficulty in ascertaining measurement errors for actual devices in the field is very real, however. Very often the weights used in commercial state estimators are chosen and modified to achieve consistent algorithmic convergence, and not based upon careful calculation of expected measurement error variances.

To measure the impact of PMU location on entropy reduction, in sequence one PMU was added to each bus, and the change on entropy was estimated using (28). At the same time, the percentage reduction in the expected variance of the phase angle error at the PMU bus was calculated by observing the change in the corresponding diagonal entry of the state error covariance matrix G^{-1}.

The calculation results are shown as scatter plots in Exhibits 2-2, 2-3 and 2-4 for three different levels of PMU accuracy (standard deviations of 0.001, 0.1 and 1.0 degrees, respectively). Each graph has two sets of data: (1) entropy reduction vs. % bus angle variance reduction, and (2) entropy reduction vs. original case bus angle variance. For each scenario, the voltage phasor measurement error standard deviation was set equal to 0.0089 p.u., matching the assumption made for the other voltage magnitude measurements.

Several observations can be made from the results:

- Adding a PMU always reduces entropy. This is necessarily true; adding a new sensor, even one that is known to have very poor accuracy, always improves estimation with the addition of new information.[11]

[10] The measurement error range estimates for voltage magnitude and real and reactive power magnitude have been adopted from [Adi90].

[11] The assumption is that the estimation algorithm (WLS, in this case) is aware of the accuracy of the newly added sensor and weights the measurement accordingly. If the weight is too high for the actual accuracy of the meter, or if the error statistics of the meter violate the assumptions of matching a zero mean Gaussian assumptions, the inaccuracy in the estimate may be increased.

- For very accurate PMUs (standard deviation = 0.001 degrees), the state estimator is expected to eliminate nearly all (100%) of the uncertainty in the angle variance where the PMU is located. This is entirely expected, of course.

- The reduction in total state uncertainty as measured by the reduction in Shannon entropy is decreased as the PMU accuracy is decreased.

- For accurate PMUs (0.1 degrees), the relationship between local uncertainty reduction (angle variance reduction) and total system state uncertainty reduction (entropy reduction) by PMU is nearly monotonic. Even for inaccurate PMUs (1.0 degrees), the relationship shows a positive correlation.

- For extremely accurate (0.001 degrees) and very accurate (0.1 degrees) PMUs, the sites (buses) where PMU addition results in maximal entropy reduction correlate very strongly to the buses where the estimated system without PMUs exhibits the largest expected phase angle variance. This is seen by observing that in Exhibits 2-2 and 2-3 the scatter plots of entropy reduction versus angle variance are nearly monotonic.

The observed correlations between entropy reduction and both local bus angle variance reduction and original bus angle variance are perhaps not surprising; they suggest that the effects of a newly added PMU are very local, and will significantly improve overall estimation accuracy only when the knowledge of the ‘local’ state is poor. If this hypothesis proves true in the general case (it would be good to derive appropriate theory to support the example calculations here), a simple guideline for placing PMUs to improve state estimation accuracy is suggested: place them where corresponding diagonal entries of G^{-1} are large.

Enthusiasm for any sensor placement techniques that require numerical analysis of any matrices dependent on measurement error statistics (e.g. the gain matrix $G = H^T R_z^{-1} H$) should be tempered because most utilities and control areas do not maintain good knowledge of the observed accuracy of existing telemetry. The recommended solution is remote meter calibration (e.g. [Zho05], [Adi90]), a variant of parameter estimation that permits both measurement accuracy determination and correction.

Exhibit 2-2: Effect of PMU Placement (PMU Accuracy = 0.001 Degrees)

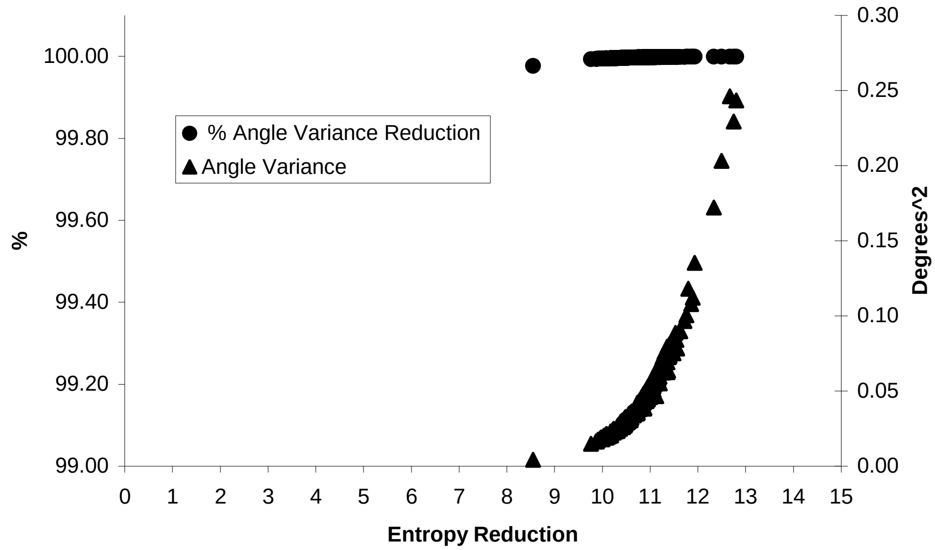


Exhibit 2-3: Effect of PMU Placement (PMU Accuracy = 0.1 Degrees)

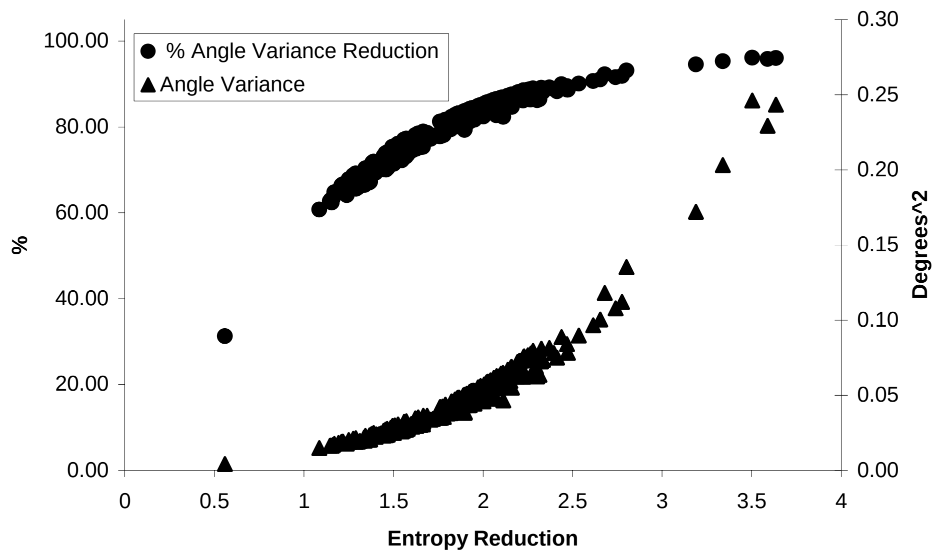
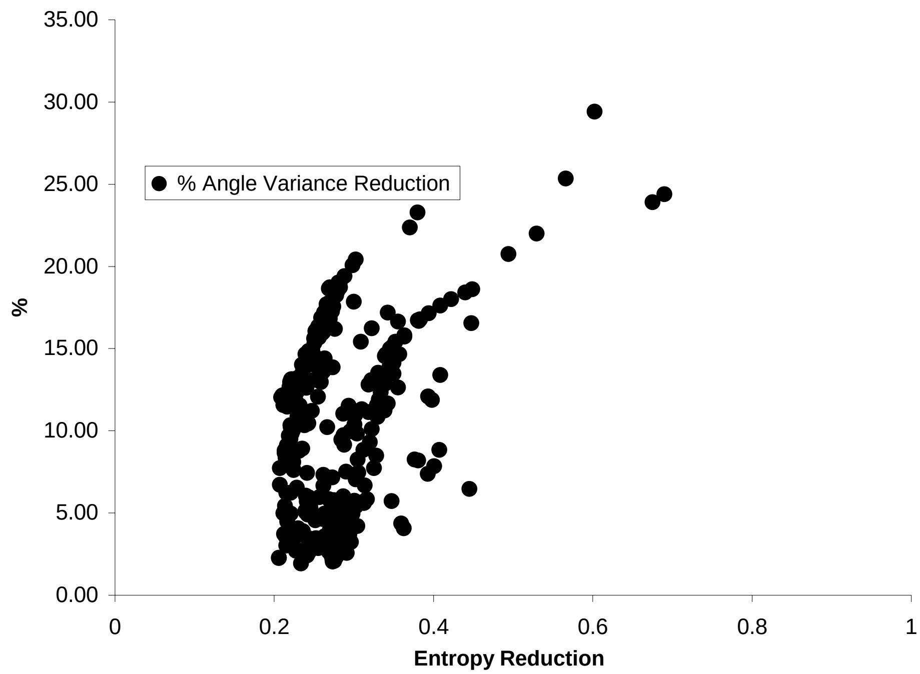
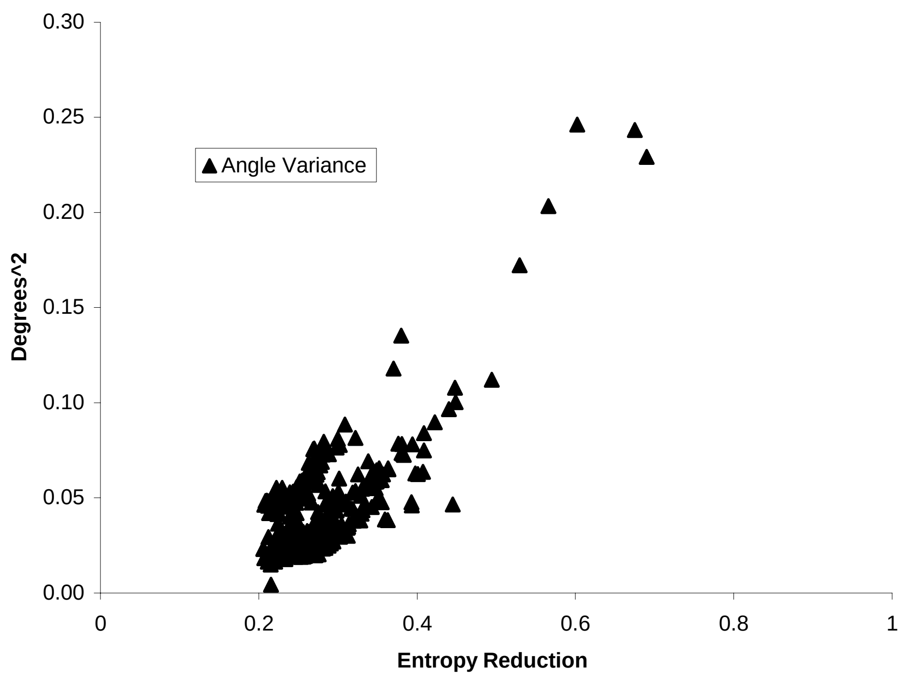


Exhibit 2-4: Effect of PMU Placement (PMU Accuracy = 1.0 Degrees)





3. Test Methodology

This section describes the process for the evaluation and analysis of the impact of phasor measurements on the quality of power system state estimation. The following assumptions are made regarding the scope of the testing:

- Electrical islanding – The tests described in this section are meant to be applied to a contiguous electrical network and are not intended to examine the effects of electrical islanding on state estimation.

- Meter placement – The location of non-phasor metering and PMUs is assumed to be known (given) prior to any metric testing. The purpose of the tests described in this section is to examine the effects of introducing phasor measurements at pre-selected locations, not to determine the best PMU locations.

- state estimator (SE) – Testing is to be conducted utilizing a given state estimator. The tests are not intended to require making any changes to the given state estimator's solution algorithms. It is acceptable to change externally exposed parameters, as long as the changes are noted in the test report.

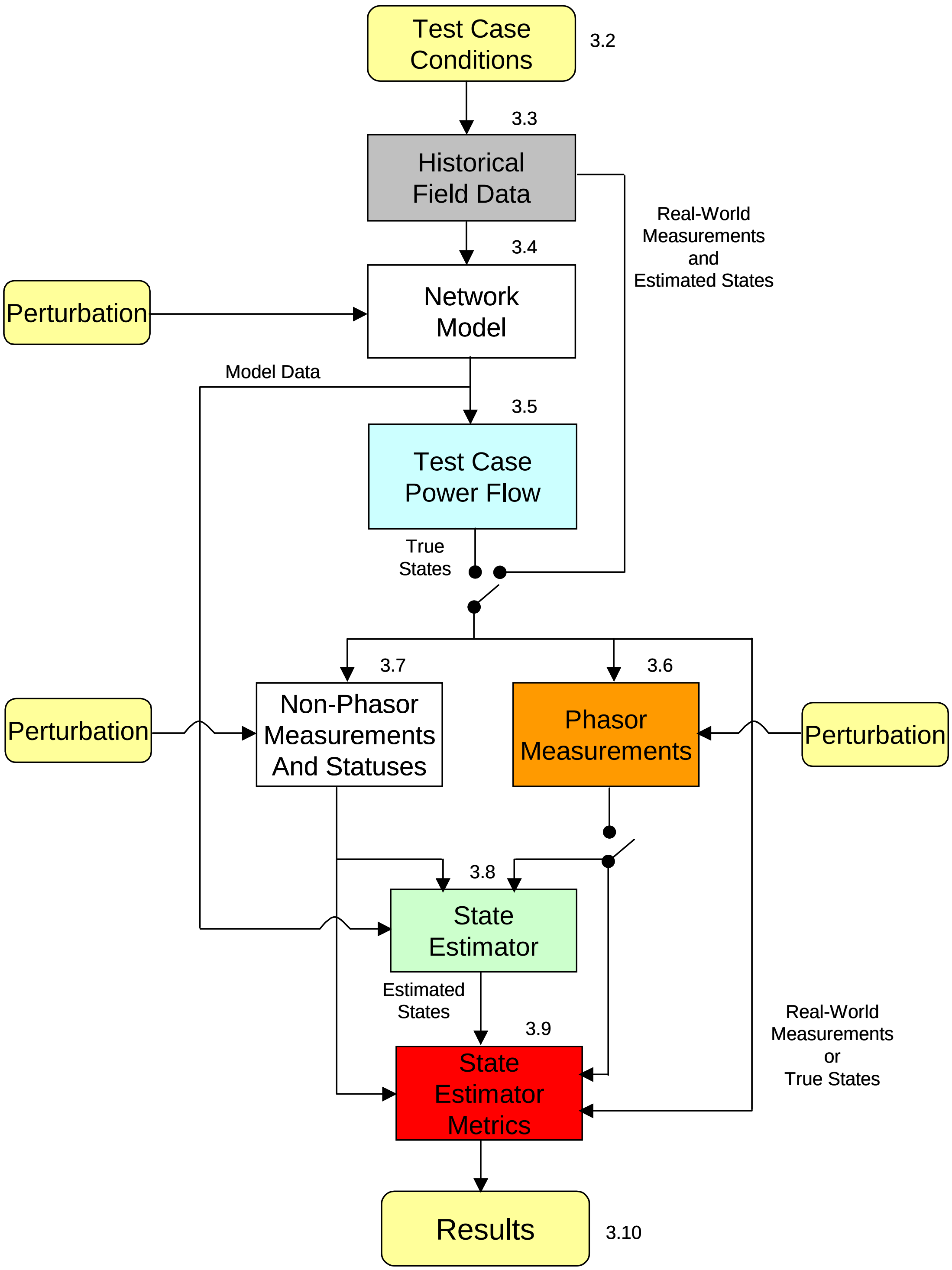
3.1 Overview

Prior to performing any tests it is necessary to establish the test case or benchmark conditions that will enable the observation of the changes caused by the introduction of phasor measurements. Test case conditions can be characterized as including the following:

- Specified power system condition, such as Summer Peak
- Combined network model representing the "internal" and "external" power system
- Load pattern for the specified power system condition
- Generation pattern for the specified power system condition
- Topology, i.e., switch statuses, that is consistent with the power system condition

Testing is intended to be "off-line" and completely under the control of the investigators. Test cases can be hypothesized or derived from actual power system conditions, and are used to start off the testing. Also, different test cases may be required depending upon the specific state estimation metric under examination. Exhibit 3-1 below provides a summary of the test process together with section references for relevant descriptions.

Exhibit 3-1: State Estimation Metrics – Test Process



3.2 Test Environment

The purpose of the "off-line" approach is to avoid any possible disruption to control center operations that could occur if tests were run "on-line". It is important that testing be conducted under controlled circumstances and that tests are repeatable. All manner of metric scenarios can be set up for detailed analysis, utilizing the given state estimator. The following can be set up in this environment:

- Hypothetical or "true states" which are not available in the field and
- Real-world historical field data

3.2.1 Test Location

The physical location of the tests is not critical and can be at the convenience of the investigators. Preferably the test location should be accessible to EIPP observers who may wish to witness the testing process and procedures.

3.2.2 Test Platform

The investigators must provide the necessary computer test platform. As a minimum, the test platform should have the following capabilities:

- Workstation with displays to show the various test inputs, outputs, and reports. The workstation should also include an office suite for word processing and spreadsheet calculations.
- Sufficiently large memory storage capability to save field data and numerous save cases
- Read/write storage media for the acquisition and dissemination of data files.
- Power Flow program that has a database compatible with the state estimator program's database
- State estimator program

3.3 Historical Field Data

Tests that are based on actual power system conditions will require obtaining information and data from an operational Energy Management System (EMS), preferably with a working state estimator. The purpose is to use data that is representative of real-world, real-time conditions. In the test process we can either use the information directly or make adjustments to the real data to reflect abnormal conditions. In the event that actual phasor measurements are not available, these measurements need to be constructed and introduced to the state estimator along with actual non-phasor measurements. This approach will allow investigators to run tests over a wide range of power system conditions and also has the advantage of being able to make repeat test runs with the same input data and operating conditions.

Refer to Section 3.3.2, Test Data Extraction, for further discussion of data processing.

3.3.1 Data Source and Retrieval

The source of the field data will need to be chosen by the investigators in collaboration with a cooperating utility organization. The intent of this project is to choose a cooperating utility entity of sufficient size and scope so that meaningful test results can be achieved. The cooperating utility entity will need to provide a sufficiently diverse sample of historical operating data together with the associated power system network model.

It is important that the data to be supplied to the investigators has been "scrubbed" to remove any sensitive power system security and electricity market information.

3.3.2 Test Data Extraction

The investigators will be responsible for extracting the required test data from the information obtained from the cooperating utility entity and converting this data to the structure and formats required by the investigators' test platform.

3.4 Power System Network Model

In order to perform the metrics testing a power system network model needs to be created. The SE model can be adapted from the test case power flow or the SE model can be built from the same database that is associated with the historical field data. Once the SE model has been constructed it can be adjusted during the testing process to simulate such things as bad model parameter data and bad topology connections.

3.4.1 Model Build and Measurement Coordination

The test platform will need to have a power system network model for performing power flow and SE studies. The investigators will be responsible for building the model from the information obtained from the cooperating utility entity, or any other source that represents the power system under investigation. An important requirement is that the model can be exported in an industry-accepted format (for example, CIM and PSS/O) for review.

The investigators will be responsible for associating the existing field measurements with their respective locations within the network model. This may require verification with the cooperating utility entity. Preliminary state estimation runs on the test platform may be required to correct any misconnected field measurements or modeling inconsistencies and to help establish good test cases.

3.4.2 Model Perturbation

The network model is used to obtain "true" states and also as the base model for SE studies. Model perturbation applies primarily to the SE studies. Once the SE model has been constructed it can be adjusted during the SE metric testing process to simulate such things as bad model parameters and bad physical topology connections.

3.5 Test Case Power Flow

In order to determine the impact of phasor measurements it is necessary to establish what is referred to as the "true state" of the power system. This is accomplished by executing a conventional power flow on the given test case conditions. The results of the test case power flow will serve as proxies, since it is not possible to obtain the true states of the real power system.

3.5.1 Test Case Preparation

Test cases can be used for arbitrary SE metric studies and for studies involving field data. Test case preparation consists of the following for any particular test case under consideration:

- Network model set up
- Establishing the bus loads
- Establishing the bus generation
- Establishing bus voltage control
- Establishing FACTS or other control settings

It is important that the test case produce a power flow solution with a high degree of convergence.

3.5.2 Power Flow Save Cases

SE metric analysis will generally require many repeat runs of identical power system conditions. Therefore, it is suggested that the test platform should be able to save the results from test case power flow and SE runs; in order to minimize the need to repeat the power flow setup and execution process.

3.6 Phasor Measurements

Although this project is primarily concerned with the application of PMU derived voltage phasor measurements, the introduction of other phasor measurements, such as current phasors, is acceptable and should be reported if they are included in the test process.

Voltage phasors are described in terms of:

- Phasor Voltage Magnitude – Analogous to non-phasor voltage measurements

- Phasor Angle – Angle of the phasor with respect to a GPS time reference

The location of the PMUs needs to be provided by the cooperating utility entity, since meter placement is not considered a function of these tests.

3.6.1 Phasor Angle Reference

Although modern PMUs are able to measure voltage and current phasor angles with respect to the GPS time reference, several factors must be accounted for to ensure accurate and consistent phase angle measurements when made available to the state estimator, including:

- The power system voltages to be measured, recognizing that the three-phase voltages are typically unbalanced and "noisy" due to harmonics and transients;
- Any Wye-Delta angular phase shifts through any local power transformer not already accounted for in the state estimator's network model;
- Characteristics of voltage (or current) transducers used by the PMU;
- The type of PMU and its processing algorithms.

Some of the above factors are generally taken into account within the PMUs, while others may need to be accounted for at a central SCADA location. An important benefit of using a state estimator in coordination with phasor measurements is an enhanced ability to detect an incorrectly installed or calibrated PMU; any improperly installed PMUs producing measurements used in state estimation will likely be identified during bad data detection.

For SE metric testing purposes a reference for the phasor angles may need to be established, for example, a specific bus in the network. The investigators together with the SE provider will have to establish the required approach and procedures, depending on the SE algorithm and the way that the phasor angles are calculated by the PMUs. This applies to actual field PMUs as well as to any "emulated" PMUs to be included in the metric tests but are not yet installed.

3.6.2 Phasor Angle Perturbation

To evaluate the impact of the PMU phasor angle measurements, the variance of the phasor angle measurements must be adjusted over some range. Variance (from true) can be caused by:

- Dither – Actual fluctuations in voltage and power system frequency
- Sample skew – Non-simultaneous PMU measurement sets

The objective is to determine the PMU impact with realistic variances as well as to set up test situations for the detection of bad phasor angles or the loss of phasor angles

3.7 Non-Phasor Measurements/Statuses

At this point in the testing process it is necessary to prepare the non-phasor measurements to be used in the investigation. Real measurements can be collected and added to the set of non-phasor measurements as well as the results of the test case power flow (which represent perfect measurements). Adjustments can be made to the power flow results to simulate metering errors, having different variances as desired for the particular test case under investigation.

This should not be viewed as a trivial task because real measurements are being simulated from a dynamic power system, with ever changing power system frequency, where a “snapshot” of measurements could range over many seconds.

3.7.1 Non-Phasor Measurement/Status Selection

The set of non-phasor measurements and statuses to be used for any test run will need to be defined and will depend on the objectives of the particular SE metric test under consideration. The sets that are chosen must be documented so that repeat runs will use the same measurements.

The desired observability and redundancy characteristics must also be considered when defining the desired non-phasor measurement/status set.

3.7.2 Non-Phasor Measurement/Status Perturbation

Tests will require adjustments to the measurement variances and the introduction of bad data. The following types of perturbations need to be considered and tested:

- Noise with a normal distribution and a given measurement variance
- Abnormally high value
- Abnormally low values
- Offset or bias
- Loss of data
- Measurement inconsistency with associated network switch status

3.8 State Estimator

This project is concerned primarily with the impact of phasor measurements upon the state estimation process, and not the relative efficacy of alternative state estimation algorithms. Therefore, we treat the state estimator as a tool. Obviously different SEs (as with tools) will exhibit different characteristics.

Thus, the investigators will need to identify and record the characteristics and idiosyncrasies of the particular SE that was used during the course of testing. This is especially important since testing could involve commercially different SEs and possibly SEs under development. The impact of phasor measurements on SE "flat-start" and "tracking" modes is also an important consideration.

3.8.1 State Estimator Metric Test Case Executions

Referring to Exhibit 3-1, the upper switch is to choose the use of the test case (i.e., true) power flow results or real-world data; the lower switch is to choose an execution of the SE with or without phasor measurements.

The SE metric test procedure, based on comparisons with true values, can be summarized as follows for any applicable metric:

Step 1 Choose the power system condition (e.g., summer peak) for the test.

Step 2 Execute the test case power flow in order to obtain the "true" states and flows. Save these true conditions without adjustment.

Step 3 Select the measurement set and apply hypothetical error characteristics (i.e., variances) to the true values in order to produce realistic measurements

Step 4 Apply the desired perturbations

Step 5 Set the adjustable SE execution parameters

Step 6 Execute SE without the phasor measurements and record the results

Step 7 Execute SE with phasor measurements and record the results

Step 8 Repeat Steps 1 thru 7 for the desired range of system conditions and test scenarios

The SE metric test procedure, based on comparisons with field values, can be summarized as follows for any applicable metric:

Step 1 Choose the power system condition (e.g., summer peak) for the test.

Step 2 Retrieve the historical real-world data.

Step 3 Select the measurement set and establish its known error characteristics (i.e., variances)

Step 4 Apply the desired perturbations

Step 5 Set the adjustable SE execution parameters

Step 6 Execute SE without the phasor measurements and record the results

Step 7 Execute SE with phasor measurements and record the results

Step 8 Repeat Steps 1 thru 7 for the desired range of system conditions and test scenarios

3.8.2 State Estimator Save Cases

It is important to be able to save the inputs and results from the various test case scenarios since many of the error characteristics will be of a statistical nature and can change with subsequent runs. The SE needs to be executed with exactly the same error characteristics and measurements, with and without the phasor measurements.

Also upon review, the reviewers may request the re-execution of specific runs under changed conditions.

3.9 State Estimation Metrics

The SE metrics that have been selected for testing should provide a good indication of how the SE will be influenced by phasor measurements.

3.9.1 Recommended State Estimation Metrics

Table 3-1 lists the metrics recommended in Section 2, the equation numbers where the metrics are defined, and comments to help the investigator use the metrics appropriately.

3.9.2 Results Comparison

For each SE metric test, the requirement is to compare the results of the SE with and without the introduction of phasor measurements. The purpose is to determine the impact of phasor measurements on the "quality" of the estimated states. The following are suggested approaches for comparing SE metric test results:

- *Raw* – report the raw scores
- *Differences* – report the difference in terms of: (After - Before)
- *Ratios* – report the ratio in terms of: (After - Before)/Before

The approach to use (raw, difference, ratio, or combination) will depend on the type of SE metric and also which approach will provide the reader with the most meaningful comparisons.

3.10 Reporting

Each of the SE metric tests is to be reported for EIPP review and should provide the following information:

- Description of the power system that was used as the test bed

- Description of the test platform
- Description of the phasor measurements used in the tests
- Description of the non-phasor measurements used in the tests
- For each SE metric test:
  - Description of metric test
  - Results of SE metric test without phasor measurements
  - Results of SE metric test with phasor measurements
  - Impact of phasor measurements on state estimation

Table 3-1: Testing Notes for Recommended Metrics

Metric	Equation	Usage Comments
$M_{Pmismatch}$	(1)	These accuracy metrics should be reported as a pair, and are applicable in both real-time and test environments. Recommended reporting units are MW and MVar, respectively.
$M_{Qmismatch}$	(2)	
$Macc_{Sflow}$	(6)	These accuracy metrics are valid only in a test environment. Recommended units of measure are MVA, and per unit, respectively.
$Macc_V$	(7)	
$Mconv_{obj}$	(12)	These metrics are most useful in a real-time setting, where metric statistics (e.g. average, worst case, histograms) are reported after observing the estimator for a significant elapsed time (e.g. one week, one month). When used to compare the effects of a change to the estimation system such as the addition of PMU measurements, a sequence of archived cases can be used.
$Mconv_V$	(13)	
$Mconv_\theta$	(14)	
$Mmeas_\sigma(p, a)$	(17)	Use of this metric for bad data identification requires many cases to be generated, as for each pair of parameters *(p,a)* a statistically significant number of cases must be generated and tested. The method used to generate the 'bad' measurements, including choice of location, should be well documented.
$Mtop(p)$	(21)	The added difficulty in using this metric is that each of the perturbed cases starts with a breaker (not bus/branch) model of the network. For each integer p for which the testing is performed, a significantly significant number of cases is required. The method used to choose topology errors should be well documented.
Mobs	(22)	Assuming the number of unobservable buses is readily available from the state estimation software, use of this metric is straightforward for both real-time and test environments.

4. Recommendations

This document has reported on the mechanism by which the impact of phasor measurements on state estimator results can be measured. State estimator metrics that provide measures of accuracy, performance, robustness, and completeness have been identified. A process for testing these metrics has also been developed. Now that this foundation has been established, the next step is to actually test a sample set of state estimators presently being used by utilities who also have PMUs installed on their electric systems.

It is suggested that the state estimator vendors be contacted and each invited to propose at least one utility that can participate with the test. It would be desirable to have at least one utility from each interconnection, i.e., Eastern, Western, and Texas. It is important to keep in mind that the test will only measure the impact of phasor measurements on the state estimator results and will not be used to evaluate the vendors' state estimator products. To insure that the tests be carried out properly and that the intent of the tests be maintained at all times, it is also suggested that an independent test monitor be chosen who can coordinate the testing activities, supervise the test set-ups, monitor the actual testing, and assist with interpreting and documenting the test results.

If it is determined that PMUs do have a significant impact on the results of state estimation, then it would be important to proceed with two additional activities. The first would be review methods for determining the optimal placement of PMUs in a utility's power system and the second would be to perform a benefit/cost analysis to determine the worth of PMU installation.

This work on the impact of phasor measurements on state estimator results will have an important influence on the reliable and economic operation of electric power systems.

Appendix A – Bibliography

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Appendix B – Terminology

Terminology

This brief section provides additional explanation for some of the terminology used in the previous sections.

True state – The real or actual condition of the power system. It is not feasible to continuously determine the true state of all the variables in a power system – it would take international standards calibrated measurement devices. Thus, we accept estimates of the electrical variables based on commercially available measuring devices.

State Estimator (SE) – The mathematical algorithm that is used to estimate the true state of the power system, using available and contrived (or pseudo) measurements applied to a model of the power system.

Measurement error – The difference between the measurement and the true state of a specified variable. It is common to assume that measurement errors are independent and random with a zero mean normal distribution In practice, however, measurement errors will be correlated for devices that share common components, such as potential or current transformers.

Measurement residual – The difference between the measurement and the estimated value of a specified variable. The estimate of the measurements is a function of the estimate of the state vector, *i.e.* $z^{est} = h(\hat{x})$. Mathematically, the measurement residual (vector) r is expressed as

$$r = z^{meas} - z^{est}(\hat{x})$$

Weighted measurement residual – The measurement residual divided by the measurement's standard deviation. Mathematically, the weighted measurement residual is expressed as follows for a single measurement denoted by the subscript i:

$$r_i / \sigma_i = (z^{meas} - z^{est}(\hat{x})) / \sigma_i$$

Injection – The net real and reactive power (i.e., generation minus load) that is connected to a network bus. The general industry sign convention is that the injection is "positive" when the generation at the bus is greater than the load at the bus.

Zero Injection Bus – This is a network bus that has no connected generators or load.

Network Flow – The net real and reactive power flow from a bus into the network elements (branches and shunts to ground) that are connected to the bus. The general industry sign convention is that the network flow is "positive" for power flowing from the bus and into the network. Calculated network flows are derived from the solved (computed) complex bus voltages and the network model.

Injection Mismatch – This mismatch applies to state estimation for a specified bus in the network and is defined as (estimated Injection – estimated Network Flow).

Norm – A convenient mathematical shorthand expression related to "length" or "distance". The following vector definitions apply to this report:

General Norm: $\|Y\|_p = \left(\sum_p |Y_i|^p \right)^{1/p}$ for p ≥ 1

Euclidian Norm: $\|Y\|_2 = \left(\sum_i |Y_i|^2 \right)^{1/2}$ for p = 2 (also the 2-norm)

Infinity Norm: $\|Y\|_\infty = \max_i |Y_i|$

State estimator objective function – The generally accepted approach to state estimation is to minimize the objective function J defined as the sum of the squares of the weighted measurement residuals:

$$J(\hat{x}) = \sum_{j=1}^{m} \left(\frac{r_j}{\sigma_j} \right)^2 = \sum_{j=1}^{m} \left(\frac{z_j - h_j(\hat{x})}{\sigma_j} \right)^2$$

State variables – Any set of variables the knowledge of which uniquely defines the current operating characteristics of the power system. The most commonly defined state variables are the bus voltage magnitudes and their associated vector phase angles. The state estimator is so-called because it produces an accurate estimate of the state variables.

State Estimator Gain Matrix – The most common definition of the gain matrix (or information matrix) G of the state estimator is defined in terms of the Jacobian matrix H of the measurement functions and the covariance matrix R_z of the measurements. The covariance matrix R_x of the state variables is equal to the inverse of the gain matrix:

$$G = H^T R_z^{-1} H$$

$$R_x = G^{-1}$$

Entropy of a 1-dimensional normal distribution – Entropy indicates the lack of knowledge of the variable. Thus, the goal of state estimation is to minimize the entropy of the state variables. Shannon's definition of entropy for a discrete random variable can be extended to cover continuous random variables. For continuous random variables entropy is sometimes referred to as "differential entropy". The entropy of a single variable having a normal (Gaussian) distribution with a standard deviation of σ is given as follows:

$$H = \tfrac{1}{2}\ln\left(2\pi e \sigma^2\right)$$

Entropy of an n-dimensional normal distribution – For state estimation purposes the entropy of interest is the entropy of $\hat{x}$, the *n*-element vector describing the state estimate. If the state estimate is described by a multivariate distribution with covariance R_x (see State Estimator Gain Matrix, above), the entropy of the state estimate is given by:

$$\begin{aligned} H &= \tfrac{1}{2} n \ln\left(2\pi e \left|\det\left(R_x\right)\right|^{1/n}\right) \\ &= \tfrac{1}{2} n\left[1 + \ln(2\pi)\right] + \tfrac{1}{2}\ln\left|\det\left(R_x\right)\right| \\ &= \tfrac{1}{2} n\left[1 + \ln(2\pi)\right] + \tfrac{1}{2}\ln\left|\det\left(G^{-1}\right)\right| \end{aligned}$$

Matrix Triangularization – A square matrix, such as the gain matrix *G* of state estimation, can be decomposed into the product of two "triangular" matrices *L* and *U*.

$$G = LU$$

L is a lower triangular square matrix whose main diagonal terms are equal to "one" and whose terms above the main diagonal are equal to "zero". The determinant of *L* is equal to "one".

U is an upper triangular square matrix whose main diagonal terms are "non-zero" and whose terms below the main diagonal are equal to "zero". The determinant of *U* is equal to the product of the main diagonal terms. Thus,

$$det\ G = det\ L\ det\ U = det\ U$$